[20] R. K. Otnes and L. Enochson, *Digital Time Series Analysis*, New York: Wiley, 1972.
[21] V. H. MacDonald and P. M. Schultheiss, "Optimum passive bearing estimation," *J. Acoust. Soc. Amer.*, vol. 46, pp. 37–43, 1969.
[22] G. C. Carter, C. H. Knapp, and A. H. Nuttall, "Estimation of the magnitude-squared coherence function via overlapped fast Fourier transfrom processing," *IEEE Trans. Audio Electroacoust.*, vol. AU-21, pp. 337–344, Aug. 1973.
[23] D. Signori, J. L. Freeh, and C. Stradling, personal communication.
[24] B. V. Hamon and E. J. Hannan, "Spectral estimation of time delay for dispersive and non-dispersive systems," *J. Royal Stat. Soc. Ser. C (Appl. Statist.)*, vol. 23, pp. 134–142, 1974.

Signal Analysis by Homomorphic Prediction

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Abstract—**Two commonly used signal analysis techniques are linear prediction and homomorphic filtering. Each has particular advantages and limitations. This paper considers several ways of combining these methods to capitalize on the advantages of both. The resulting techniques, referred to collectively as homomorphic prediction, are potentially useful for pole-zero modeling and inverse filtering of mixed phase signals. Two of these techniques are illustrated by means of synthetic examples.**

Introduction

Two classes of signal processing techniques which have been applied to a variety of problems are homomorphic filtering or cepstral analysis [1], [2] and linear prediction or predictive deconvolution [3]-[5]. Separately, each has particular advantages and limitations. It appears possible, however, to combine them into new methods of analysis which embody the advantages of both. In this paper we discuss several ways of doing this.

Linear prediction is directed primarily at modeling a signal as the response of an all-pole system. Its chief advantage over other identification methods is that for signals well matched to the model it provides an accurate representation with a small number of easily calculated parameters. However, in situations where spectral zeros are important linear prediction is less satisfactory. Furthermore, it assumes that the signal is either minimum phase or maximum phase, but not mixed phase. Thus, for example, linear prediction has been highly successful for speech coding [3], [5], [6] since an all-pole minimum phase representation is often adequate for this purpose. It has also been applied in the analysis of seismic data, although limited by the fact that such data often involve a significant mixed phase component.

Homomorphic filtering was developed as a general method of separating signals which have been nonadditively combined. It has been used in speech analysis to estimate vocal tract transfer characteristics [7]-[9] and is currently being evaluated in seismic data processing as a way of isolating the impulse response of the earth's crust from the source function [10]-[12]. Unlike linear prediction, homomorphic analysis is not a parametric technique and does not presuppose a specific model. Therefore, it is effective on a wide class of signals, including those which are mixed phase and those characterized by both poles and zeros. However, the absence of an underlying model also means that homomorphic analysis does not exploit as much structure in a signal as does linear prediction. Thus, it may be far less efficient than an appropriate parametric technique when dealing with highly structured data.

The basic strategy for combining linear prediction with cepstral analysis is to use homomorphic processing to transform a general signal into one or more other signals whose structures are consistent with the assumptions of linear prediction. In this way the generality of homomorphic analysis is combined with the efficiency of linear prediction. In the next section we briefly review some of the properties of homomorphic analysis that suggest this approach. We then discuss several specific ways of combining the two techniques.

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Homomorphic Signal Processing

Homomorphic signal processing is based on the transformation of a signal $x(n)$ as depicted in Fig. 1. Letting $\hat{X}(z)$ and $X(z)$ denote the z transforms of $\hat{x}(n)$ and $x(n)$, the system $D_*[\cdot]$ is defined by the relation

$$\hat{X}(z) = \log X(z) \tag{1}$$

where the complex logarithm of $X(z)$ is appropriately defined

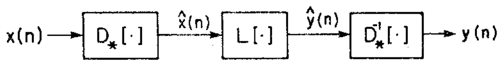

Fig. 1. Canonic homomorphic processor.

[2]. The system $L[\cdot]$ is a linear system and the system $D_*^{-1}[\cdot]$ is the inverse of $D_*[\cdot]$. The signal $\hat{x}(n)$ is commonly referred to as the complex cepstrum. There are a number of properties of the complex cepstrum that are particularly useful, and these have been discussed elsewhere [1], [2]. Of particular interest for this paper are the following.

1) With $x(n)$ expressed as the convolution of its minimum phase and maximum phase components, denoted as $x_{\min}(n)$ and $x_{\max}(n)$, respectively,

$$\hat{x}(n) = \hat{x}_{\min}(n) + \hat{x}_{\max}(n).$$

Furthermore, $\hat{x}_{\min}(n)$ is zero for $n < 0$ and $\hat{x}_{\max}(n)$ is zero for $n > 0$. Thus, the complex cepstrum provides a means for factoring a signal into its minimum phase and maximum phase components. Specifically, by choosing the linear system in Fig. 1 such that

$$\hat{y}(n) = \begin{cases} 0 & n < 0 \\ \frac{1}{2}\hat{x}(0) & n = 0 \\ \hat{x}(n) & n > 0 \end{cases}$$

the output $y(n)$ will be equal to $x_{\min}(n)$. Alternatively, choosing the linear system such that

$$\hat{y}(n) = \begin{cases} \hat{x}(n) & n < 0 \\ \frac{1}{2}\hat{x}(0) & n = 0 \\ 0 & n > 0 \end{cases}$$

will result in an output $y(n)$ equal to $x_{\max}(n)$.

2) A mixed phase signal can be converted to a minimum phase signal with the same spectral magnitude by choosing the linear system such that

$$\hat{y}(n) = \begin{cases} 0 & n < 0 \\ \hat{x}(0) & n = 0 \\ \hat{x}(n) + \hat{x}(-n) & n > 0. \end{cases} \tag{2}$$

The resulting output $y(n)$ will then be a minimum phase signal with the same spectral magnitude as $x(n)$. An alternative and equivalent procedure for obtaining $y(n)$ is to use as the input to the system of Fig. 1 $\phi_{xx}(n)$, the autocorrelation of $x(n)$. In that case, (2) can be expressed as

$$\hat{y}(n) = \begin{cases} 0 & n < 0 \\ \frac{1}{2}\hat{\phi}_{xx}(0) & n = 0 \\ \hat{\phi}_{xx}(n) & n > 0. \end{cases}$$

3) If $x(n)$ has a rational z transform, then $n\hat{x}(n)$ has a rational z transform whose poles correspond to the poles and zeros of the z transform of $x(n)$. This follows in a straightforward way from (1) by noting that the z transform of $n\hat{x}(n)$ is given by $-z d\hat{X}(z)/dz$ and

$$-z\frac{d}{dz}\hat{X}(z) = -z\frac{1}{X(z)}\frac{dX(z)}{dz}.$$

With $X(z)$ of the form $N(z)/D(z)$, then

$$-z\frac{d}{dz}\hat{X}(z) = -z\frac{D(z)N'(z) - N(z)D'(z)}{N(z)D(z)} \tag{3}$$

where the prime denotes differentiation with respect to z. More specifically, let $X(z)$ be written in the form

$$X(z) = \frac{Az^r \prod_{k=1}^{M_i}(1 - a_k z^{-1}) \prod_{k=1}^{M_0}(1 - b_k z)}{\prod_{k=1}^{P_i}(1 - c_k z^{-1}) \prod_{k=1}^{P_0}(1 - d_k z)} \tag{4}$$

where $|a_k|$, $|b_k|$, $|c_k|$, and $|d_k|$ are all less than unity. The a_k's and c_k's correspond to zeros and poles inside the unit circle in the z plane, representing the minimum phase component of $x(n)$, and the $(1/b_k)$'s and $(1/d_k)$'s correspond to zeros and poles outside the unit circle and thus represent the maximum phase component of $x(n)$. In order for the complex cepstrum of $x(n)$ to exist, the time origin of $x(n)$ is chosen so that $r = 0$. With $X(z)$ in the form of (4) and with $r = 0$,

$$\log X(z) = \log A + \sum_{k=1}^{M_i}\log(1 - a_k z^{-1}) + \sum_{k=1}^{M_0}\log(1 - b_k z) - \sum_{k=1}^{P_i}\log(1 - c_k z^{-1}) - \sum_{k=1}^{P_0}\log(1 - d_k z)$$

and $-z\, d\hat{X}(z)/dz$ can be expressed as

$$-z\frac{d\hat{X}(z)}{dz} = \sum_{k=1}^{M_i}\frac{-a_k z^{-1}}{1 - a_k z^{-1}} + \sum_{k=1}^{P_i}\frac{c_k z^{-1}}{1 - c_k z^{-1}} + \sum_{k=1}^{M_0}\frac{b_k z}{1 - b_k z} - \sum_{k=1}^{P_0}\frac{d_k z}{1 - d_k z}. \tag{5}$$

4) Let $x(n)$ have an irrational z transform in the form

$$X(z) = Az^r \prod_{k=1}^{N_i}(1 - a_k z^{-1})^{\alpha_k} \prod_{k=1}^{N_0}(1 - b_k z)^{\beta_k} \tag{6}$$

where $|a_k|$ and $|b_k|$ are all less than unity. If the α_k and β_k are all integers (positive or negative), then $X(z)$ reduces to a rational z transform as discussed in 3) above. More generally, however, we assume here that the α_k and β_k are not restricted to integer values. Again, in order for the complex cepstrum of $x(n)$ to exist the time origin of $x(n)$ is chosen so that $r = 0$.

With $X(z)$ of the form of (6) and with $r = 0$

$$\log X(z) = \log A + \sum_{k=1}^{N_i}\alpha_k \log(1 - a_k z^{-1}) + \sum_{k=1}^{N_0}\beta_k \log(1 - b_k z)$$

and

$$-z\frac{d\hat{X}(z)}{dz} = \sum_{k=1}^{N_0} \frac{\beta_k b_k z}{1 - b_k z} - \sum_{k=1}^{N_i} \frac{\alpha_k a_k z^{-1}}{1 - a_k z^{-1}}. \quad (7)$$

From (7) we observe that the linearly weighted cepstrum $n\hat{x}(n)$ still has a rational z transform, with first-order poles corresponding to each of the irrational factors in (6).

Homomorphic Prediction

Given the above properties of the complex cepstrum, we now outline several ways of using homomorphic filtering to prepare a signal for analysis by linear prediction. This set of techniques we refer to collectively as homomorphic prediction.

1) Factorization of Mixed Phase Signals: As discussed previously, one limitation of linear prediction is its restriction to minimum or maximum phase signals. However, by exploiting property 1), a mixed phase signal can be factored into its minimum and maximum phase components. These can then be analyzed separately using linear prediction.

2) Cepstral Prediction: A second limitation of linear prediction is its inability to locate spectral zeros. From property 3), however, we note that the poles of $n\hat{x}(n)$ correspond to the poles and zeros of the original signal. Thus, an all-pole analysis of $n\hat{x}(n)$ leads to a pole-zero representation of $x(n)$. This technique, in which linear prediction is applied to the cepstrum, has been referred to as cepstral prediction [13], [14]. In using this procedure, it is necessary to classify each pole of $n\hat{x}(n)$ as either a pole or a zero of the original signal. This can be done in a number of ways. One approach is to do a separate linear prediction analysis of $x(n)$ to estimate its poles and then identify the remaining poles of $n\hat{x}(n)$ as the zeros of $x(n)$. An alternate strategy is suggested by (5). Let p_i be a pole of $n\hat{x}(n)$ inside the unit circle. If it corresponds to a zero of $X(z)$ (one of the a_k), then the residue of $-zd\hat{X}(z)/dz$ at p_i is $-p_i$. On the other hand, if p_i is a pole of $X(z)$ (one of the c_k), then the residue is $+p_i$. Thus, a pole of $n\hat{x}(n)$ inside the unit circle can be classified as either a pole or zero of $x(n)$ by comparing the sign of the residue with the sign of the pole. There is a corresponding test to separate the poles and zeros of $X(z)$ outside the unit circle. A particularly effective method for evaluating these residues has been proposed by Atashroo [15]. If the original data consist of a basic pulse convolved with an impulse train, the low-time portion of the complex cepstrum corresponds to the complex cepstrum of the basic pulse. Thus, because of the properties of the complex cepstrum, the necessary deconvolution is effected automatically, and for all-pole modeling of the basic pulse the above method would be applied only to the low-time portion of the complex cepstrum.

3) Fractional Pole-Zero Modeling: On the basis of property 4) of homomorphic analysis as discussed above, linear prediction can also be combined with homomorphic analysis to model data in terms of an irrational z transform in the form of (6). In this case we first obtain the linearly weighted complex cepstrum $n\hat{x}(n)$. Pole-zero modeling of $n\hat{x}(n)$ is then carried out to obtain the parameters a_k, α_k, b_k, and β_k as indicated in (7). Again, if the original data consist of a basic pulse convolved with an impulse train, the necessary deconvolution is effected automatically because of the properties of the complex cepetrum.

4) Pole-Zero Modeling Using Homomorphic Deconvolution and Inverse Filtering: A somewhat different approach to pole-zero modeling is motivated by the existence of several generalizations of linear prediction which explicitly include spectral zeros [16]-[18]. Typically, these methods proceed by using linear prediction to estimate the poles. An inverse filter is then applied to the data to obtain a residual signal which is subsequently modeled by zeros using one of a variety of possible methods. In most of these methods the data to be analyzed must generally have the proper time registration and no linear phase component. Futhermore, if the data consist of a pulse which is to be modeled in terms of poles and zeros convolved with an impulse train an estimate of the basic pulse must first be carried out, i.e., the composite signal must first be deconvolved. If not, the analysis will generate zeros to approximate the fine structure in the spectrum introduced by the impulse train.

In order to provide the proper time registration and also implement the required deconvolution, homomorphic filtering can first be used. If it is sufficient to determine the minimum phase counterparts of the poles and zeros, the homomorphic filtering can first be applied to obtain a minimum phase signal. A minimum phase signal by definition has no linear phase component and hence is properly aligned. Alternatively, the signal can first be factored into its minimum and maximum phase elements. Because neither of these has a linear phase component, they are synchronized properly. With homomorphic filtering to obtain either a minimum phase or mixed phase signal, the required deconvolution is also implemented. Following the homomorphic filtering, one of the above methods for pole-zero modeling can be applied. This system is similar to, but represents a generalization of that described in [17].

Examples

In this section we present a number of synthetic examples to illustrate some of the above ideas. These examples are intended to be illustrative of the theoretical concepts rather than as an evaluation of the techniques.

Example 1–Inverse Filter Design for Mixed Phase Signals: The first example is concerned with the design of two sided inverse filters for mixed phase signals. The general block diagram of this technique is presented in Fig. 2.

A mixed phase signal $x(n)$ was synthesized and is depicted in Fig. 3. From its complex cepstrum, shown in Fig. 4, the minimum phase and maximum phase components $x_{\min}(n)$ and $x_{\max}(n)$ were obtained (Figs. 5 and 6, respectively). From these, two predictive filters $h_{\min}(n)$ and $h_{\max}(n)$ were designed.

Filtering the synthetic data through $h_{\min}(n)$ yields the output of Fig. 7. Note how successfully the causal predictive filter has removed $x_{\min}(n)$ leaving only the maximum phase component. The output of the inverse filter $h_{\min}(n) * h_{\max}(n)$ is depicted in Fig. 8 and it consists basically of a delayed impulse. (This delay is known *a priori* from the linear phase component that was removed in the process of cepstral computation.)

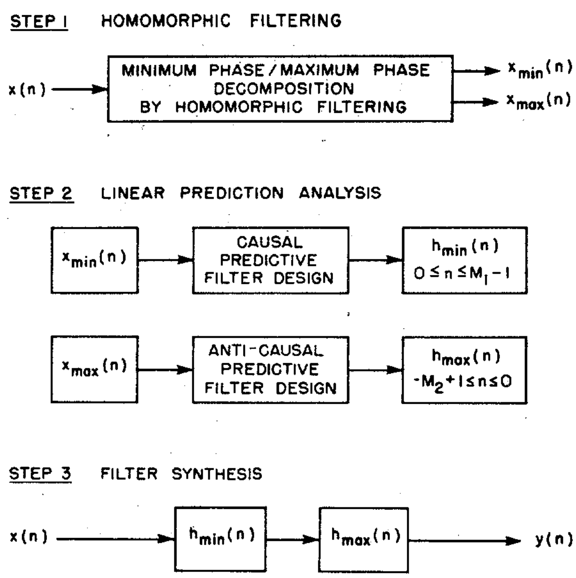

Fig. 2. Inverse filter design by homomorphic prediction.

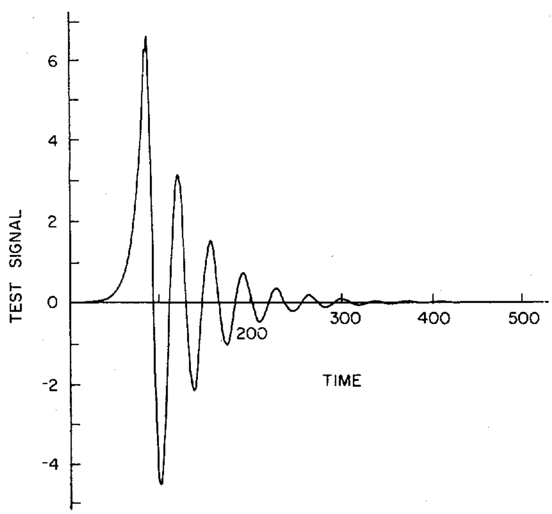

Fig. 3. Mixed phase signal $x(n)$.

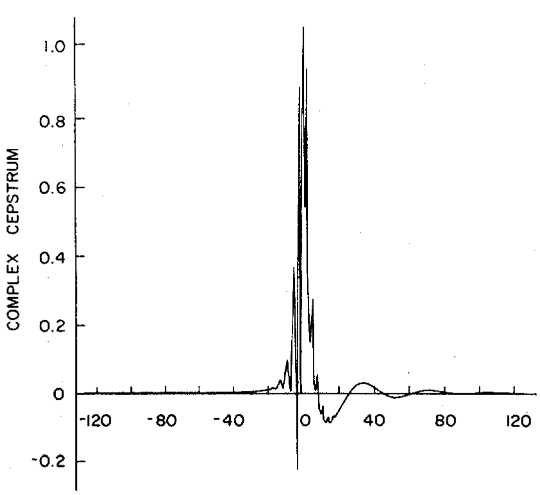

Fig. 4. Complex cepstrum of $x(n)$.

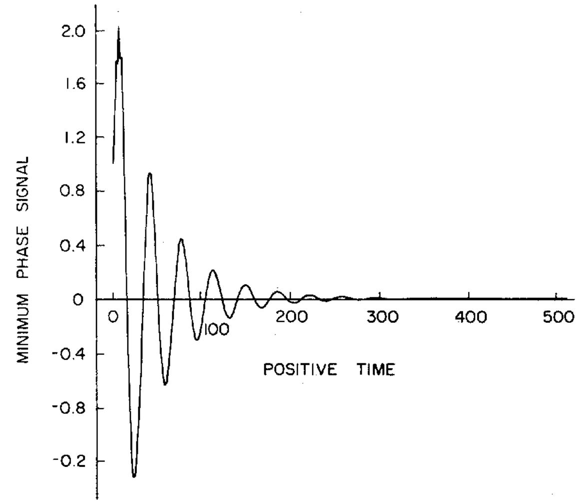

Fig. 5. Minimum phase component $x_{\min}(n)$.

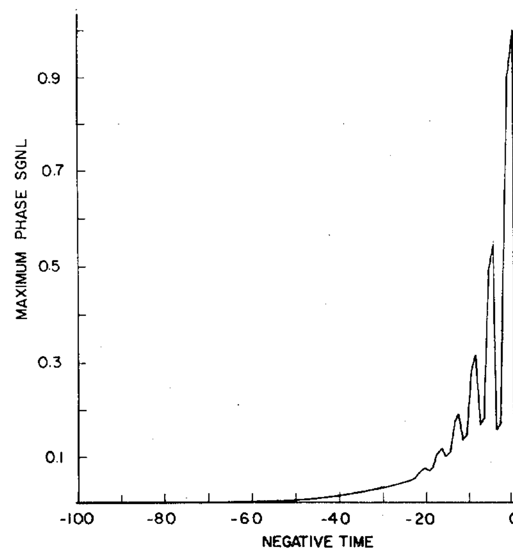

Fig. 6. Maximum phase component $x_{\max}(n)$.

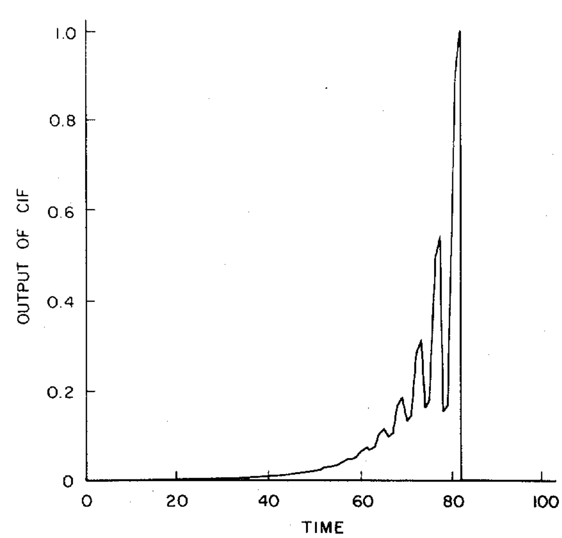

Fig. 7. Output of $h_{\min}(n)$.

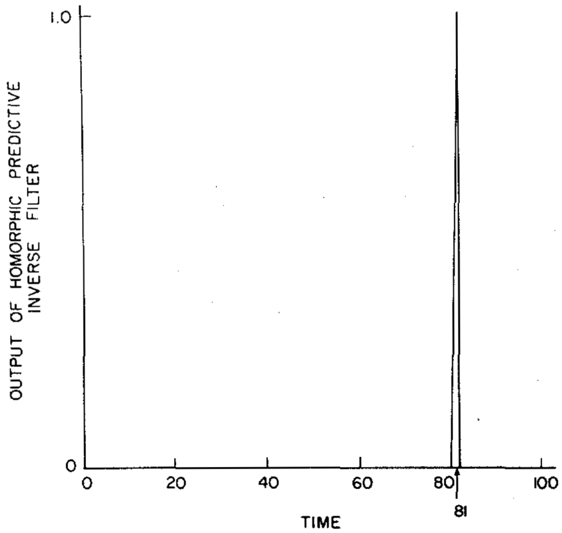

Fig. 8. Output of $h_{\min}(n) * h_{\max}(n)$.

The technique illustrated in this example is currently being evaluated for the design of inverse filters for seismic data processing. Our preliminary results indicate considerable improvement over the the use of current techniques.

Example 2–Pole-Zero Modeling Using Homomorphic Deconvolution and Inverse Filtering: As discussed above, one way to satisfy the time synchronization requirement for the pole-zero modeling of voiced speech is to use homomorphic deconvolution to obtain a minimum phase estimate of the vocal tract impulse response. In this example we demonstrate this technique with an artificial signal. Fig. 9(a) shows the impulse response of a digital filter with the following poles and zeros (assuming a 12 kHz sampling rate).

pole-zero	frequency	bandwidth
P	292	79
P	3500	100
Z	2000	200

The corresponding log magnitude spectrum is given in Fig. 10(a). If this system is excited by a periodic pulse train whose period is 100 samples (120 Hz), a typical output segment would look like that in Fig. 9(b). Let us call this sequence $\{s(n)\}$.

Figs. 9(c) and 10(c) show the result of applying homomorphic deconvolution to $\{s(n)\}$ in order to estimate the impulse response. This signal $\{v(n)\}$ was found by retaining the first 50 points of the minimum phase cepstrum of $\{s(n)\}$. A 256-point Hamming window was used to smooth $\{s(n)\}$ before computing its cepstrum.

To illustrate the use of the filtered signal $\{v(n)\}$ in pole-zero analysis, Shanks' method was used to approximate $V(z)$ with four poles and two zeros [18]. Fig. 9(d) shows the impulse response and Fig. 10(d) the frequency response of the resulting model system. Its poles and zeros are as follows.

pole-zero	frequency	bandwidth
P	291	118
P	3498	128
Z	2004	242

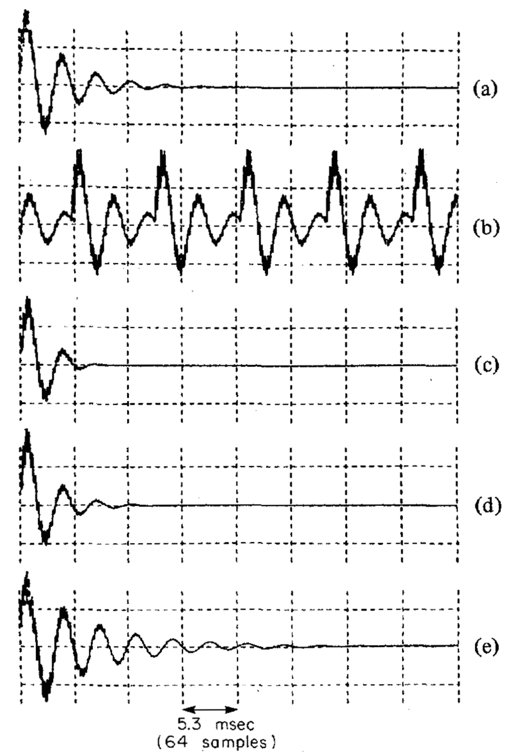

Fig. 9. Time signals for Example 2 (pole-zero modeling) showing (a) actual impulse response, (b) impulse train response, (c) homomorphic estimate of impulse response, (d) 4-pole–2-zero model of (c): Shanks' method, and (e) 6-pole model of (c): LPC.

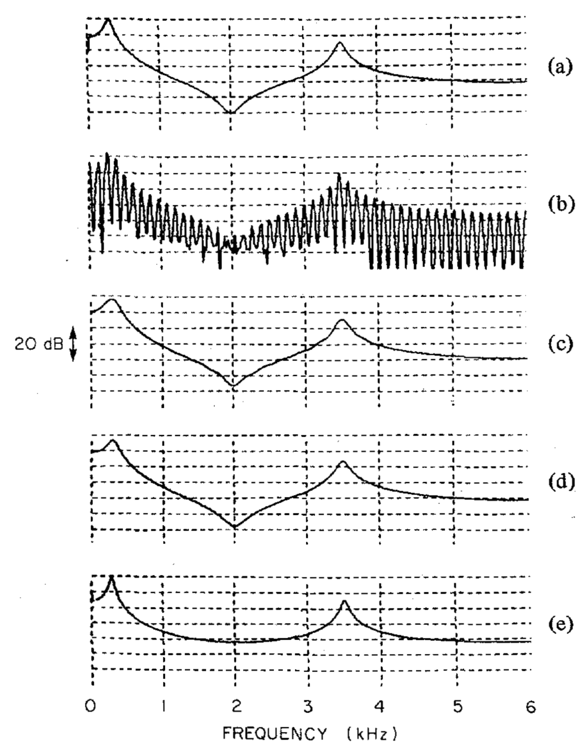

Fig. 10. Log spectra for Example 2 (pole-zero modeling) showing (a) actual spectral envelope, (b) impulse train response, (c) homomorphic estimate of spectral envelope, (d) 4-pole–2-zero model of (c): Shanks' method, and (e) 6-pole model of (c): LPC.

Finally, if six-pole linear prediction (autocorrelation method) is applied directly to the original signal $\{s(n)\}$, the frequency response of the resulting all-pole filter is that in Fig. 10(e), corresponding to the time waveform of Fig. 9(e).

At the present time we are evaluating this approach to pole-zero modeling in the context of real speech. Preliminary results indicate that the technique is considerably more reliable than direct analysis of the speech waveform.

References

[1] A. V. Oppenheim, R. W. Schafer, and T. G. Stockham, "Nonlinear filtering of multiplied and convolved signals," *Proc. IEEE*, vol. 56, pp. 1264-1291, Aug. 1968.
[2] A. V. Oppenheim and R. W. Schafer, *Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975, ch. 10.
[3] J. Makhoul, "Linear prediction: A tutorial review," *Proc. IEEE (Special Issue on Digital Signal Processing)*, vol. 63, pp. 561-580, Apr. 1975.
[4] L. Wood and S. Treitel, "Seismic signal processing," *Proc. IEEE (Special Issue on Digital Signal Processing)*, vol. 63, pp. 649-661, Apr. 1975.
[5] B. S. Atal and S. L. Hanauer, "Speech analysis and synthesis by linear prediction of the speech wave," *J. Acoust. Soc. Amer.*, vol. 50, pp. 637-655, 1971.
[6] J. D. Markel, A. H. Gray, Jr., and H. Wakita, "Linear prediction of speech–Theory and practice," Speech Commun. Res. Lab., Santa Barbara, CA, Monograph 10, Sept. 1973.
[7] A. V. Oppenheim and R. W. Schafer, "Homomorphic analysis of speech," *IEEE Trans. Audio Electroacoust.*, vol. AU-16, pp. 221-226, June 1968.
[8] A. V. Oppenheim, "Speech analysis-synthesis system based on homomorphic filtering," *J. Acoust. Soc. Amer.*, vol. 45, pp. 459-462, 1969.
[9] R. W. Schafer and L. R. Rabiner, "System for automatic analysis of voiced speech," *J. Acoust. Soc. Amer.*, vol. 47, pt. 2, pp. 634-648, 1970.
[10] T. J. Ulrych, "Application of homomorphic deconvolution to seismology," *Geophysics*, vol. 36, pp. 650-660, Aug. 1971.
[11] P. L. Stoffa, P. Buhl, and G. M. Bryan, "The application of homomorphic deconvolution to shallow-water marine seismology–Part I: Models," *Geophysics*, vol. 39, pp. 401-416, Aug. 1974.
[12] P. Buhl, P. L. Stoffa, and G. M. Bryan, "The application of homomorphic deconvolution to shallow-water marine seismology–Part II: Real data," *Geophysics*, vol. 39, pp. 417-426, Aug. 1974.
[13] A. V. Oppenheim and J. Tribolet, "Pole-zero modeling using cepstral prediction," Res. Lab. Electron., Mass. Inst. Technol., Cambridge, Quarterly Progress Rep. 111, pp. 157-159, Oct. 15, 1973.
[14] J. Tribolet, "Identification of linear discrete systems with applications to speech processing," S.M. thesis, Dep. Elec. Eng., Mass. Inst. Technol., Cambridge, Jan. 1974.
[15] M. A. Atashroo, "Cepstral prediction analysis of digital waveforms," in *Proc. IEEE 1975 Region Six Conf.*
[16] G. E. Kopec, "Speech analysis by homomorphic prediction," S.M. Thesis, Dep. Elec. Eng., Mass. Inst. Technol., Cambridge, Nov. 1975.
[17] J. Makhoul, "Selective linear prediction and analysis-by-synthesis in speech analysis," Bolt Beranek and Newman, Inc., Cambridge, MA, Rep. 2578, Apr. 1974.
[18] J. L. Shanks, "Recursion filters for digital processing," *Geophysics*, vol. 32, pp. 33-51, Feb. 1967.
[19] C. Weinstein and A. V. Oppenheim, "Predictive coding in a homomorphic vocoder," *IEEE Trans. Audio Electroacoust.*, vol. AU-19, pp. 243-248, Sept. 1971.

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Pseudorandom Arrays Generated by Two-Dimensional Digital Filtering

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Abstract—**This correspondence describes properties of multilevel pseudorandom arrays obtained by two-dimensional digital filtering of binary pseudonoise (PN) arrays derived from maximal-length linear binary sequences.**

Introduction

The methods of linear system theory may be generalized to two dimensions, with application to optical signal processing systems [1] and image processing [2]. In such systems random and pseudorandom arrays may be expected to play an analogous role to the important role that random and pseudorandom sequences play in one-dimensional systems.

One-dimensional periodic binary sequences having a two-level autocorrelation function (pseudonoise (PN) sequences) are well known and have many applications [3], [4]. The most useful are the binary maximal-length linear sequences (m-sequences) of period $(2^n - 1)$ digits, which exist for all integers, n. Multilevel pseudorandom sequences [5], [6] and signals for simulating analog noise [7] may be produced by appropriate filtering of these binary sequences. This correspondence discusses the analogous two-dimensional filtering of pseudorandom binary arrays.

PN Arrays

An array $a(i,j)$ of infinite extent in both dimensions is doubly periodic with periods p and q if

$$a(i+p,j) = a(i,j+q) = a(i,j) \qquad \text{for all } i,j.$$

The array thus comprises $(p \times q)$ blocks repeated along both dimensions. A class of two-dimensional doubly periodic arrays having a two-level autocorrelation function (PN arrays) which are analogous to m-sequences has been described by Gordon [8] and by Calabro and Wolf [9].

If $(2^n - 1)$ has relatively prime factors p, q, a binary PN array of this type having periods p and q can be constructed.[1]

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[1] There are (one-dimensional) binary PN sequences with periods other than $2^n - 1$ (e.g., quadratic residue sequences, twin-prime sequences, etc.) [3], and similarly other types of binary PN arrays can be constructed for which $p \cdot q \neq 2^n - 1$.