Equations (8), (9), (10), and (12) are the desired design equations for digital spectrum analysis using the I_0-sinh window. To design a window for prescribed values of R and Δf requires simply the computation of α from (10), the subsequent computation of N using (9), and finally, the computation of $w(n)$ using the subroutine [3] for I_0 [ ]. With these equations it is no more difficult to use this window for spectrum analysis than to use one of the multitude of other windows that are commonly used [6]. The I_0-sinh window, however, is much more flexible than almost all the other windows; and, using (12) and the other equations given here, it is convenient to explore the tradeoffs between record length, spectral resolution, and leakage in digital spectrum analysis.

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Reduction of Quantization Noise In PCM Speech Coding

JAE S. LIM AND ALAN V. OPPENHEIM

Abstract—**A new technique to reduce the effect of quantization noise in PCM speech coding is proposed. The procedure consists of using dither noise to ensure that the quantization errors can be modeled as additive signal-independent noise, and then reducing this noise through the use of a noise reduction system. The procedure is illustrated with examples.**

I. Introduction

Wide-band speech coding systems typically rely on general waveform coding techniques [1], [2], such as instantaneous quantization (PCM), for which the step size is fixed, any of a variety of forms of adaptive quantization, and differential schemes, such as delta modulation, adaptive delta modulation, continuously variable slope delta modulation, etc. Instantaneous quantization with fixed step sizes has the advantage that the quantizer and coder are particularly straightforward, although as the step size increases, the quantization effects become severe. In this correspondence we propose a new technique for reducing the effect of quantization noise in PCM speech coding. As described in Section II, the procedure consists of using dither noise to ensure that the quantization errors can be modeled as additive signal-independent noise, and then reducing this noise through the use of any of a variety of noise reduction systems. The procedure is illustrated in Section III.

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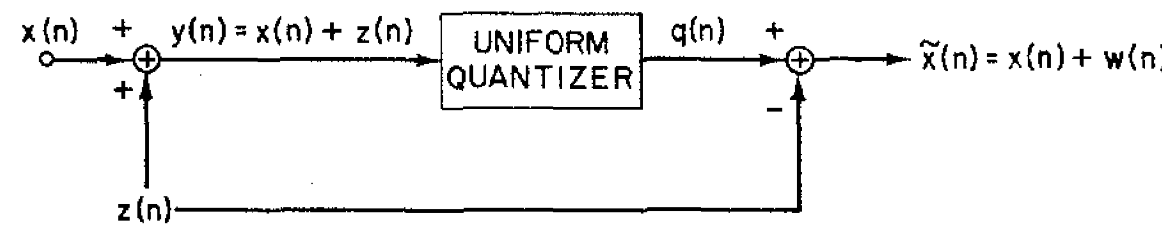

Fig. 1. System for decorrelating quantization errors in a uniform quantizer using pseudorandom noise.

II. Procedure for Quantization Noise Reduction

For linear quantization, in which the step sizes are fixed and equal, if the input signal fluctuates sufficiently and if the quantization step size is small enough, the quantization error can be modeled as additive uniformly-distributed white noise that is statistically uncorrelated with the signal [3]. When the step size becomes sufficiently large, the quantization error becomes signal dependent. For such cases, however, it is well known [4]–[6] that through the use of "dither" noise, as illustrated in Fig. 1, the quantization error can be replaced by white noise which is uniformly distributed and statistically independent of the signal. In this system, $z(n)$ is a pseudorandom uniformly-distributed white-noise sequence, with the probability density function

$$p_{z(n)}(z) = \frac{1}{\Delta} \quad \text{for} \quad -\frac{\Delta}{2} \leqslant z \leqslant \frac{\Delta}{2}$$

$$0 \text{ otherwise} \tag{1}$$

where Δ is the quantizer step size. A similar technique [4] has been used in image processing to remove the contouring effect evident in uniform image quantization.

For nonuniform fixed quantization with dither noise the quantizer can be represented conceptually in the form of Fig. 2 where $F[\cdot]$ is a specified nonlinearity. In the implementation of a nonuniform quantizer (in the form of Fig. 2) the quantization effects are additive white noise prior to the nonlinearity $F^{-1}[\cdot]$.

In this correspondence we propose a system to reduce the effects of quantization noise for general nonuniform quantization systems of the form of Fig. 2. The system exploits the fact that, in the system of Fig. 2, the quantization error is at an intermediate stage as additive white signal-independent noise. Recently, a number of procedures have been proposed and developed [7] for the enhancement of speech degraded by additive uncorrelated background noise. Our procedure for reducing the effects of quantization then corresponds to inserting a noise reduction system into the system of Fig. 2, as indicated in Fig. 3. The sequences $\hat{x}(n)$ and $\hat{s}(n)$ in the figure represent an estimate of $x(n)$ and $s(n)$. A system similar to Fig. 3 was considered by Lim [8] with encouraging results in reducing the quantization noise in the context of PCM image coding.

III. Examples

We illustrate the characteristics of the system of Fig. 3 through an example in which uniform quantization is assumed so that the nonlinearity $F[\cdot]$ is eliminated. The noise reduction system used is the revised linearized maximum *a posteriori* estimation (RLMAP) speech enhancement system [9], [10]. In this system speech is modeled on a short-time basis

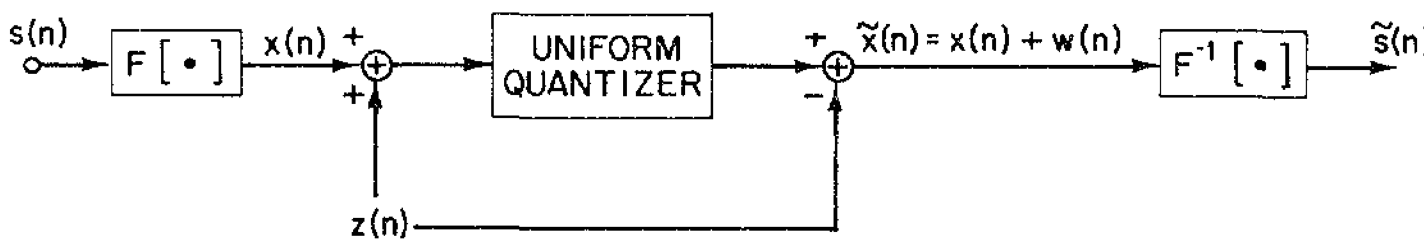

Fig. 2. System for decorrelating quantization errors in a nonuniform fixed-level quantizer using pseudorandom noise.

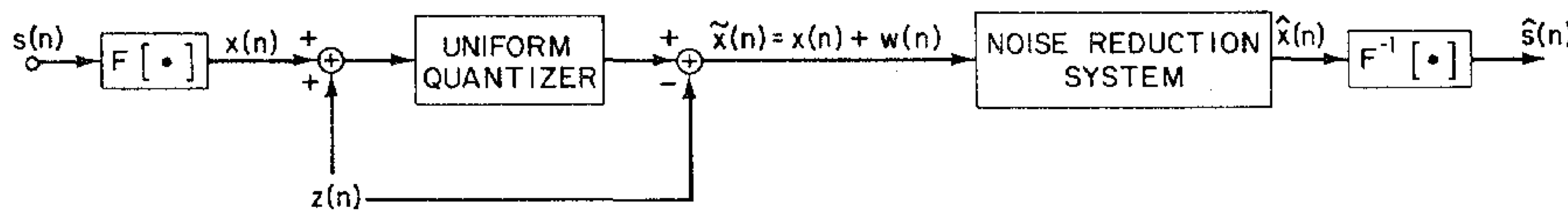

Fig. 3. A system for the reduction of quantization noise in PCM speech coding with a nonuniform fixed-level quantizer.

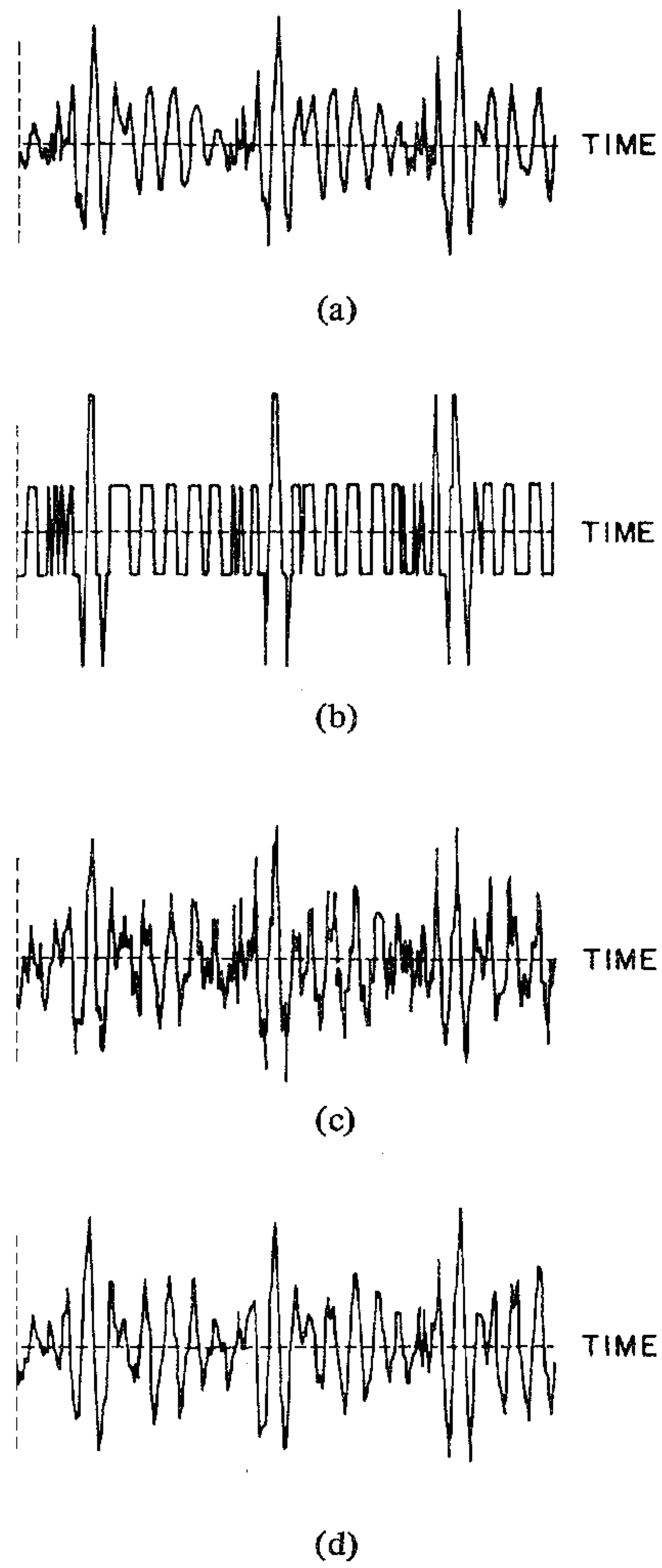

Fig. 4. (a) A segment of voiced speech. (b) Speech waveform of Fig. 4(a) coded by a PCM system with a 2-bit uniform quantizer. (c) Result of using pseudorandom noise for the speech waveform of Fig. 4(a) in a PCM system with a 2-bit uniform quantizer. (d) Result of the application of a noise reduction system to the waveform of Fig. 4(c).

as the response of an all-pole system excited by white Gaussian noise. With this model of speech, the all pole parameters are estimated by maximum *a posteriori* (MAP) estimation procedure from the noisy speech. This involves solving sets of linear equations in an iterative manner. The estimated all-pole parameters are then used to design a filter that filters the noisy speech to reduce additive noise. Further details on the RLMAP speech enhancement system can be found in [9], [10].

In Fig. 4(a) is shown a segment of noise-free voiced speech. In Fig. 4(b) is shown the speech waveform of Fig. 4(a) coded by a PCM system with a 2-bit uniform quantizer. The effect of quantization is quite visible in the staircase shape of the waveform. In Fig. 4(c) is shown the result of adding and then subtracting dither noise in a PCM system with a 2-bit uniform quantizer. In Fig. 4(d) is shown the result obtained by applying the RLMAP speech enhancement system to the waveform in Fig. 4(c).

Figs. 5 and 6 illustrate the procedure as carried out on the sentence "line up at the screen door" spoken by a male speaker. In Fig. 5(a) is shown the spectrogram of the original sentence. Fig. 5(b), (c), and (d) correspond to spectrograms of the speech in Fig. 5(a) based on PCM coding, PCM coding with addition and subtraction of dither noise, and PCM coding

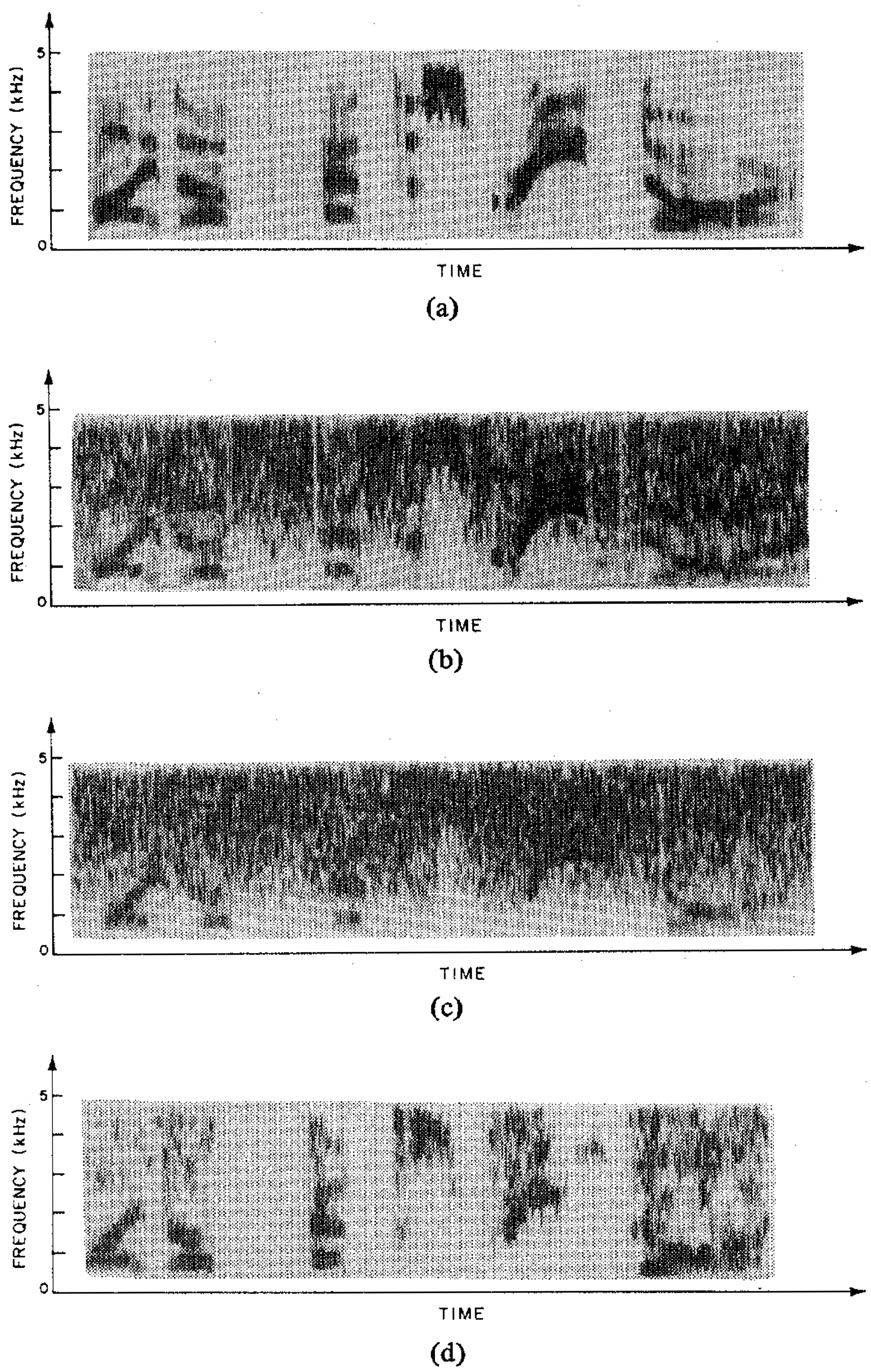

Fig. 5. (a) Spectrogram of an English sentence "line up at the screen door" spoken by a male speaker. (b) Spectrogram of speech in Fig. 5(a) coded by a PCM system with a 2-bit uniform quantizer. (c) Spectrogram of speech in Fig. 5(a) obtained by using pseudorandom noise in a PCM system with a 2-bit uniform quantizer. (d) Spectrogram of speech obtained by applying a noise reduction system to speech in Fig. 5(c).

with addition and subtraction of dither noise followed by the RLMAP speech enhancement system, respectively. The PCM coding system used in Fig. 5 is again a 2-bit uniform quantizer. Fig. 6(a), (b), and (c) is essentially the same as Fig. 5(b), (c), and (d), with the difference that the PCM system used is a 4-bit uniform quantizer. It is clear from the examples that the quantization noise is noticeably reduced.

The perceptual benefits, such as improvement in speech intelligibility or the quality of our proposed approach to reduce the quantization noise in PCM speech coding, can only be determined by a detailed subjective test and such a study is under consideration. Based on a very informal listening with a few processed sentences using the RLMAP system, however, it appears that the specific system we have implemented has the potential to improve speech quality.

IV. Conclusions

In this correspondence we have proposed a new technique for reducing the effects of quantization noise by first decorrelating the quantization noise and then using a noise reduction system to reduce the decorrelated noise. Preliminary examples

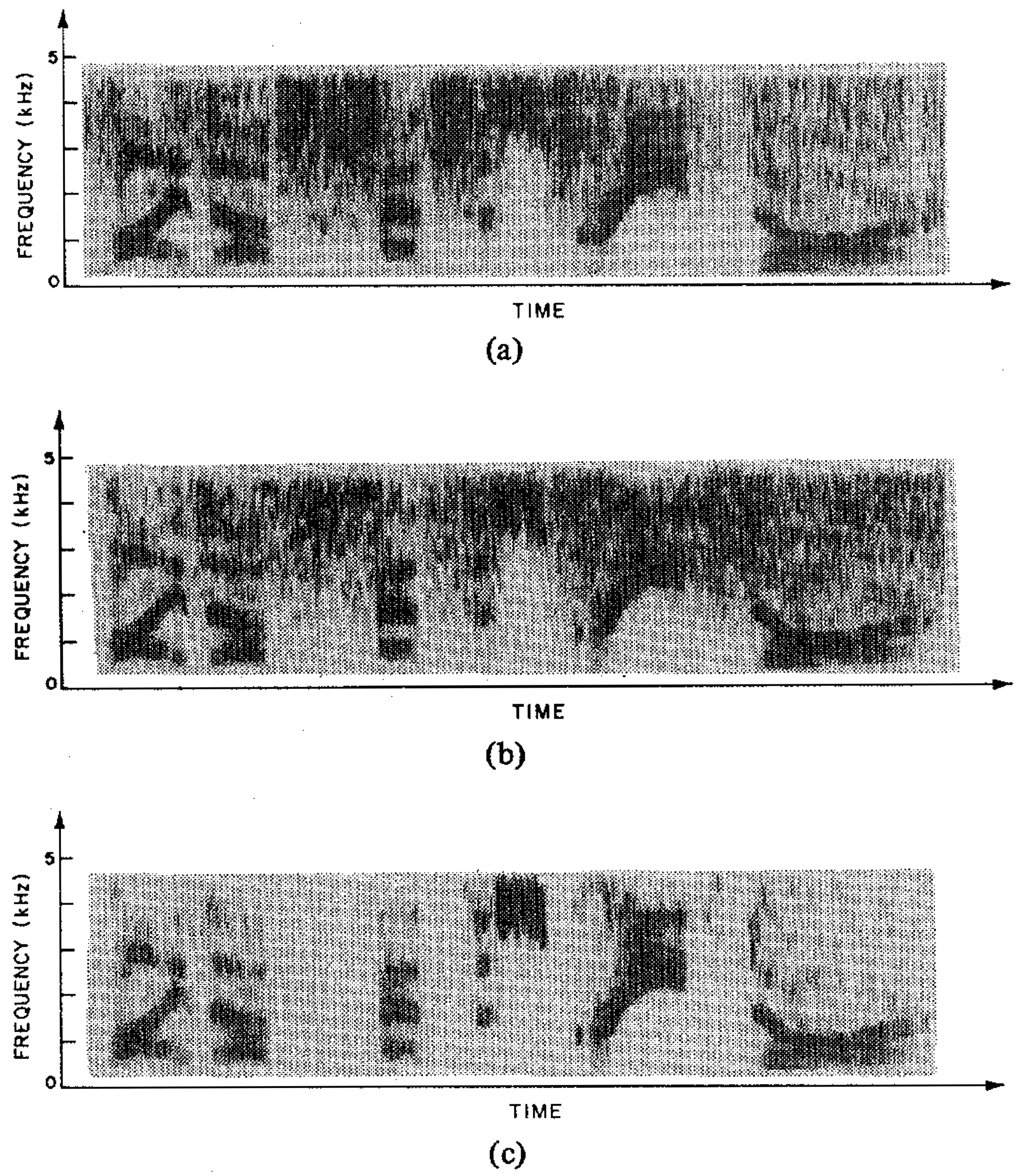

Fig. 6. (a) Spectrogram of speech in Fig. 5(a) coded by a PCM system with a 4-bit uniform quantizer. (b) Spectrogram of speech in Fig. 5(a) obtained by using pseudorandom noise in a PCM system with a 4-bit uniform quantizer. (c) Spectrogram of speech obtained by applying a noise reduction system to speech in Fig. 6(b).

based on a specific noise reduction system support the basic hypothesis and are very encouraging. While the basis for the procedure is associated with fixed nonuniform quantization, it is speculated that it can be generalized to other quantization schemes, such as differential quantization. Furthermore, it is well known that in speech enhancement systems, there is a potential benefit if the undegraded speech signal is available for preprocessing prior to insertion of the additive noise. This suggests that the system of Fig. 3 can potentially be generalized to allow for such preprocessing immediately following or prior to the first nonlinearity.

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An Adaptive Lattice Algorithm for Recursive Filters

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Abstract—**The purpose of this correspondence is to introduce an adaptive algorithm for recursive filters, which are implemented via a lattice structure. The motivation for doing so is that stability can be achieved during the adaptation process. For convenience, the corresponding algorithm is referred to as an "adaptive lattice algorithm" for recursive filters. Results pertaining to using this algorithm in a system-identification experiment are also included.**

I. Derivation of Algorithm

We consider a digital filter whose transfer function is given by

$$H(z) = \frac{a_0(n) + a_1(n)\,z^{-1} + \cdots + a_N(n)\,z^{-N}}{1 + b_1(n)\,z^{-1} + \cdots + b_N(n)\,z^{-N}} \tag{1}$$

where the $a_i(n)$ and $b_i(n)$ denote the filter coefficients (weights) at time n. This filter can be implemented in the form of a lattice with different weights $v_i(n)$ and $k_i(n)$ [1] (see Fig. 1), which is stable if the lattice coefficients $k_i(n)$ satisfy the condition [1], [2]

$$|k_i(n)| < 1; \qquad 1 \leqslant i \leqslant N. \tag{2}$$

The input-output of the above filter at time n can be expressed as

$$y(n) = \sum_{i=0}^{N} v_i(n)\,B_i(n) \tag{3}$$

where

$$\begin{aligned} B_i(n) &= B_{i-1}(n) + k_i(n)\,F_{i-1}(n); & i &= 1, 2, \cdots, N \\ F_i(n) &= F_{i+1}(n) - k_{i+1}(n)\,B_i(n-1); & i &= N, \cdots, 1 \\ F_N(n) &= x(n) \end{aligned} \tag{4}$$

and

$$B_0(n) = F_0(n).$$

If $d(n)$ denotes the desired output at time n, then the corresponding instantaneous MSE is

$$e^2(n) = [d(n) - y(n)]^2. \tag{5}$$

Use of the method of steepest descent to minimize (5), with respect $v_i(n)$ and $k_i(n)$, leads to the following equations for updating them:

$$\begin{aligned} v_i(n+1) &= v_i(n) + \mu_1 e(n)\,\phi_i(n) \\ k_i(n+1) &= k_i(n) + \mu_2 e(n)\,\psi_i(n) \end{aligned} \tag{6}$$

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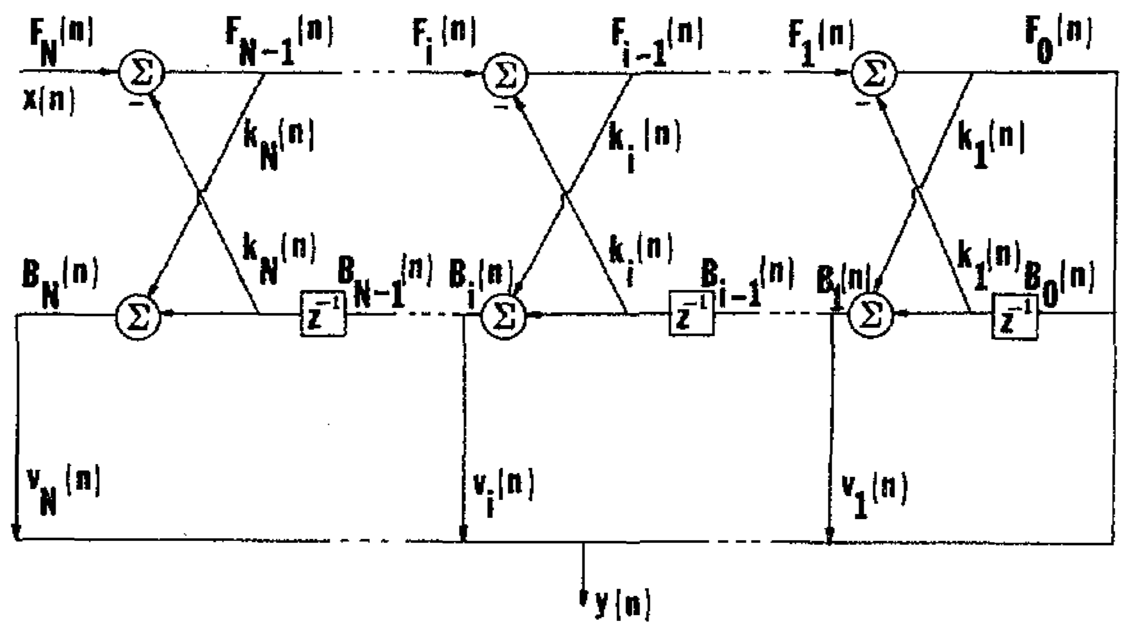

Fig. 1. Implementation of $H(z)$ in (1) using the lattice structure.

where μ_1 and μ_2 are convergence constants

$$\phi_i(n) = \frac{\partial y(n)}{\partial v_i(n)} \qquad \text{and} \qquad \psi_i(n) = \frac{\partial y(n)}{\partial k_i(n)}.$$

Computation of $\phi_i(n)$ and $\psi_i(n)$

It is necessary to compute the gradient terms $\phi_i(n)$ and $\psi_i(n)$ in (7) via a recursive relation [3]. Now, from (3) and (4), it follows that

$$\phi_i(n) = B_i(n)$$

and

$$\psi_i(n) = \sum_{j=0}^{N} v_j(n)\,\beta_{i,j}(n); \qquad i = 0, 1, \cdots, N-1. \tag{7}$$

Here,

$$\beta_{i,j}(n) = \frac{\partial B_j(n)}{\partial k_i(n)} = \begin{cases} \beta_{i,j-1}(n-1) + k_j(n)\,\alpha_{i,j-1}(n); & i \neq j \\ \beta_{i,j-1}(n-1) + k_j(n)\,\alpha_{i,j-1}(n) + F_{j-1}(n); & i = j \end{cases}$$

where

$$\alpha_{i,j}(n) = \frac{\partial F_j(n)}{\partial k_i(n)} = \begin{cases} \beta_{i,j}(n); & j = 0, \\ \alpha_{i,j+1}(n) - k_{j+1}(n)\,\beta_{i,j}(n-1); & j \neq i+1 \\ \alpha_{i,j+1}(n) - k_{j+1}(n)\,\beta_{i,j}(n-1) - B_j(n-1); & j = i+1. \end{cases}$$

Normalized Convergence Constant

In order to maintain the same adaptive time constant and misadjustment at each stage in the lattice, the convergence constant is normalized by the power level at each stage [4]. Thus, (6) is rewritten as

$$\begin{aligned} v_i(n+1) &= v_i(n) + \frac{\mu}{\sigma_i^2(n)}\,e(n)\,\phi_i(n) \\ k_i(n+1) &= k_i(n) + \frac{\mu}{\gamma_i^2(n)}\,e(n)\,\psi_i(n) \end{aligned} \tag{8}$$

where μ is the convergence constant and $\sigma_i^2(n)$ and $\gamma_i^2(n)$ are estimates of the power at the ith stage for $v_i(n)$ and $k_i(n)$, respectively, and computed as follows [4]:

$$\begin{aligned} \sigma_i^2(n) &= \rho\sigma_i^2(n-1) + (1-\rho)\,B_i^2(n) \\ \gamma_i^2(n) &= \rho\sigma_i^2(n-1) + (1-\rho)\,[B_{i-1}^2(n-1) + F_i^2(n)] \end{aligned} \tag{9}$$