Transmitter Antenna Array Broadcasting with Side Information

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Abstract — **Performance limits are derived for wireless transmission of common information from a multiple-element antenna array to multiple receivers when the relevant fading channel coefficients are all known at the transmitter. We characterize the efficient frontier of operating points and show that even when the number of receivers is larger than the number of antennas, significant performance improvements can be obtained by tailoring the transmission strategy to the realized channel.**

I. Introduction and Channel Model

We consider transmission of a common message x to L receivers simultaneously using an M-element transmitter antenna array given complete channel knowledge at the transmitter. Expected SNR is used as a performance metric, which characterizes the performance of typical uncoded systems.

In the equivalent baseband model for the narrowband system of interest, the received signal takes the form

$$Y_l = \sum_{i=1}^{M} \alpha_{i,l} h_i x + V_l, \qquad l = 1, 2, \dots, L. \tag{1}$$

where the h_i are appropriately selected complex antenna weights; $\sum_i h_i^2 \leq 1$ so the transmitted power is $\mathcal{E}_s$. The independent complex Gaussian channel coefficients $\alpha_{i,l}$ represent the Rayleigh fading between antenna element i and receiver l and have a common variance σ_α^2. The additive white Gaussian receiver noise and interference terms V_l are independent and have variance N_0. Collecting the l equations represented by (1) into a single equation in matrix form yields

$$Y = Ahx + V \tag{2}$$

II. Transmitter Operating Characteristic

The conflicting objectives inherent in transmitting to multiple receivers are naturally captured in what we define as the "transmitter operating characteristic" for the channel. For $L = 2$ receivers, this is the set of received SNR pairs $(\text{SNR}_1, \text{SNR}_2)$ for which SNR_1 is maximized subject to the constraint $\text{SNR}_2 \geq \eta$. This frontier is equivalently traced out by maximizing

$$w_1 \text{SNR}_1 + w_2 \text{SNR}_2 \tag{3}$$

as the weights w_1 and w_2 are varied, and can be calculated by observing that (3) achieves its maximum value when h is set as the eigenvector corresponding to the largest eigenvalue of $A^H W A$ where $W = \text{diag}(w_1, w_2)$. Note that various operating points of interest can be readily determined from the resulting frontier: for instance, that which maximizes SNR to a particular user, that which maximizes the minimum of the two receiver SNR's, and that which maximizes the average of the two SNR's.

III. Maximizing the Sum of SNR's

Maximizing the average SNR among all users is particularly tractable, and additionally maximizes the minimum time-averaged SNR if receivers undergo slow ergodic variations in their channel coefficients.

In general, average SNR performance improves as the number of receivers L decreases, which we characterize by defining an "SNR enhancement factor"

$$\gamma = \frac{\text{SNR with finite receiver population}}{\text{average SNR for infinite receiver population}}, \tag{4}$$

where the denominator is equivalent to ignoring channel information and equals $\sigma_\alpha^2 \mathcal{E}_s / N_0$. The expected value of γ increases from 1 to M as L decreases from ∞ to 1, reaching its maximum value when transmitting to a single recipient ($L = 1$) by a beamforming strategy with $h \propto \alpha^*$. Note that it is in general not possible to beamform simultaneously to multiple recipients.

When the number of antennas M and the number of receivers L are moderate to large, we can take advantage of asymptotic properties of the Wishart-distributed matrix $A^H A$. When M and L approach infinity in such a way that the ratio M/L of transmitter antenna elements per receiver approaches a positive constant, then the largest eigenvalue of $A^H A$ can be shown to converge almost surely [1], resulting in an average SNR enhancement per receiver of

$$\gamma_{\text{avg}} \xrightarrow{a.s.} \left(1 + \sqrt{\frac{M}{L}}\right)^2. \tag{5}$$

Monte-Carlo simulations show that expected per-receiver SNR is approximated reasonably closely by (5) when M and L are of moderate size. When more accurate statistics are required for smaller systems, probability density functions may be analytically derived [1, 2]. Although this particular strategy of maximizing average SNR is not optimized for minimizing outage probability (the probability that SNR drops below some prescribed threshold) for a particular receiver, outage probability is neverless significantly improved over a strategy that does not that channel information into account.

More detailed development and discussion of these and related results is contained in [2].

References

[1] A. Edelman, "Eigenvalues and Condition Numbers of Random Matrices," PhD thesis, Massachusetts Institute of Technology, May 1989.

[2] A. Narula, M. J. Lopez, M. D. Trott, and G. W. Wornell, "Efficient Use of Side Information in Multiple- Antenna Data Transmission over Fading Channels," to appear in *IEEE J. Select. Areas Commun.*

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