

# Successively Structured CEO Problems

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**Abstract** — **Successive encoding strategies are developed for some prototype sensor network problems based on generalized Wyner-Ziv encoding. Their performance is analyzed for quadratic-Gaussian versions of both a new serially structured CEO problem as well as the classical CEO problem.**

## I. INTRODUCTION

In the CEO problem [1],  $L$  sensors (or ‘agents’) observe independently-corrupted versions of a common source  $x^n$ . They send messages over separate rate-constrained links to a central data fusion site (the CEO), which produces a source estimate. In the serial version, the  $L$  agents are ordered and communicate—one to the next—over rate-constrained links, the final agent in the chain being termed the CEO. In both problems, the objective is to minimize the distortion in the source estimate produced by the CEO subject to the rate constraints in the problem. Our treatment of these problems is based on a generalization of the Wyner-Ziv problem.

In the Wyner-Ziv problem [7] an  $n$ -length independent identically distributed source vector  $x^n$  is observed at the encoder, while the decoder observes  $y^n$ , which is generated from  $x^n$  according to the memoryless channel law  $p(y|x)$ . In a ‘noisy’ Wyner-Ziv problem, instead of  $x^n$ , the encoder observes  $z^n$ , which is generated from  $x^n$  according to the memoryless channel law  $p(z|x)$ , where  $y^n$  and  $z^n$  are conditionally independent given  $x^n$ . It can be shown that the rate distortion function for finite-alphabet sources is  $R(d) = \min[I(z; u) - I(y; u)]$ , where the minimization is over all choices of the random variable  $u$  and over all functions  $f(\cdot, \cdot)$  such that  $E[D(x, f(y, u))] \leq d$ , with  $D(x^n, \hat{x}^n) = \frac{1}{n} \sum_{i=1}^n D(x_i, \hat{x}_i)$  denoting the distortion measure, and with  $u$  satisfying the Markov relations  $u \leftrightarrow z \leftrightarrow x$  and  $u \leftrightarrow x \leftrightarrow y$ . The derivation [4] generalizes [3, 7], accommodating the lack of perfect source observations via application of the Markov Lemma. The channel coding dual, “writing on dirty paper wearing foggy glasses,” is developed in [2, 4]. In the quadratic-Gaussian case in which  $d$  denotes mean-square distortion (MSD), we obtain

$$R(d) = 0.5 \log [(\sigma_{x|y}^2 - \sigma_{x|y,z}^2)/(d - \sigma_{x|y,z}^2)].$$

## II. SUCCESSIVE ENCODING FOR CEO PROBLEMS

The noisy Wyner-Ziv problem arises in the serial CEO problem as follows. Each agent has a decoder and an encoder. Its decoder uses its own sensor measurement as side information when decoding the message sent by the previous agent in the chain, and then combines the two together to produce a source estimate. Its encoder then sends a message that best reduces what it thinks the next agent’s estimation error will be. Thus the encoder for agent  $l-1$  and the decoder for agent  $l$  form a noisy Wyner-Ziv pair.

For the quadratic-Gaussian version of this problem, it can be shown that the following region is achievable [4]

$$d_l = \frac{N_l d_{l-1}}{N_l + d_{l-1}} + \sigma_{x|y_l}^2 \frac{\left(1 - \frac{d_{l-1}}{\sigma_x^2}\right)}{\left(1 + \frac{d_{l-1}}{N_l}\right)} 2^{-2R_{l-1}}, \quad l = 1, 2, \dots$$

where  $d_l$  is the MSD achieved by the  $l$ th agent’s decoder,  $d_0 = \sigma_x^2$  with  $\sigma_x^2$  denoting the variance of each of the source elements,  $y_l$  is the measurement available at the  $l$ th agent’s sensor with  $N_l$  denoting the corresponding noise variance, and  $R_{l-1}$  is the communication rate between agents  $l-1$  and  $l$ .

It is also straightforward to show that if the rates on the links in the problem are all identical ( $R_i = R$ ), then the MSD saturates at  $\Theta(2^{-R})$ , but that it suffices for the rates to grow according to  $R_i \sim \Theta(\log l)$  for the MSD to stay within any fixed dB gap of the  $R \rightarrow \infty$  bound  $\sigma_{x|y_1, \dots, y_l}^2$  for all  $l$  [4].

By ordering the agents, an alternative solution to the classical CEO problem can similarly be developed. In particular, each agent encodes its measurement taking into account what the decoder will receive from previous agents, and the decoder decodes the agents’ transmissions in order, using the previously decoded messages as side information in decoding the current message.

In the quadratic-Gaussian version of this problem [5, 6], this encoding strategy yields

$$d_l = \frac{N_l d_{l-1}}{N_l + d_{l-1}} + \frac{d_{l-1}^2}{N_l + d_{l-1}} 2^{-2R_l}, \quad l = 1, 2, \dots$$

where  $d_l$ ,  $N_l$  and  $d_0 = \sigma_x^2$  are as before, but  $R_l$  is now the communication rate between the  $l$ th agent and the CEO. Subject to a sum-rate constraint, the resulting distortion is the minimum possible for both the  $L \rightarrow \infty$  and  $L = 2$  cases [4].

These techniques also play an analogous role in coding for the relay channel, as discussed in [2, 4].

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