SINGLE SENSOR ACTIVE NOISE CANCELLATION BASED ON THE EM ALGORITHM

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ABSTRACT

In this paper we develop an approach to active noise cancellation using a single microphone. The noise field is modeled as a stochastic process and a time adaptive algorithm based on a modification of the block Estimate Maximize (EM) algorithm is used to adaptively estimate the parameters of this process. Based on these parameter estimates a cancelling signal is generated. The algorithm developed is evaluated with recordings of aircraft noise, and has been implemented in real time with a single AT&T DSP32C chip.

1 INTRODUCTION

Unwanted acoustic noise is a by-product of many industrial processes and systems. Active noise cancellation(ANC) is an approach to noise cancellation in which a secondary noise source is introduced which destructively interferes with the unwanted noise. Many ANC systems utilize two or more sensors. An upstream measurement of the noise field is used as input to an adaptive filter which is adjusted based on measurement of the downstream cnacelled field [4,5]. A difficulty with multiple sensor systems is that there is typically some feedback from the cancelling noise source to the upstream sensor. This can be taken into account in the adaptive filter[1] but instability can result. Consequently it is often preferable to use only a single sensor.

In this paper we present a new ANC system based on a modification of the Estimate Maximize (EM) algorithm [2]. Our approach is to model the noise field at any point as a stochastic process and use an adaptive algorithm to estimate the characteristics of this process. These estimated characteristics are then used to predict the future values of the noise field, and a cancelling source is used to generate a second acoustic field equal in magnitude and opposite in phase to these estimated values. If the prediction is accurate, the two fields will interfere destructively and cancel each other. The resulting system only requires one microphone, and therefore avoids the acoustic feedback problem inherent in many multiple-microphone systems.

The proposed algorithm derives from a time-domain formulation of the EM algorithm. In this formulation, the EM algorithm corresponds to iteratively applying a Kalman smoother to data blocks. By replacing the Kalman smoother with a Kalman filter and the iteration index by a time index, the block algorithm is converted to a sequential algorithm [3].

The algorithm developed in this paper, was used to cancel the noise generated by a helicopter, a propeller airplane, and a jet airplane, in real-time simulations using an AT&T DSP32C Digital Signal Processor chip. Assuming that the cancelling speaker and the noise source were three centimeters apart, and that the transfer function between the cancelling speaker and the microphone is a pure delay the algorithm was able to attenuate the overall noise power by 47dB, 45dB, and 33dB respectively.

2 MODEL SPECIFICATIONS

A generic single-microphone ANC system is depicted in Figure 1. In general the transfer function characteristics from the output of the cancelling speaker to the output of the microphone need to be accounted for. However, if this transfer function can be accurately measured in advance and is invertible except for a propagation delay between the speaker and the microphone, the essential problem becomes one of predicting the future values of the noise field. Consequently, in the remainder of this paper we concentrate on the idealized problem represented in figure 2. In this figure, the noise $s(t)$ to be cancelled is modeled as a stochastic process consisting of white noise $\sigma_s u(t)$ filtered by a transfer

This work was done in part in the RLE Digital Signal Processing Group, MIT. This work has been supported in part by the Defense Advanced Research Projects Agency monitored by ONR under Contract No. N00014-89-J-1489, and U.S. Air Force-Office of Scientific Research under Grant No. AFOSR-91-0034.

The E-step for each iteration now corresponds to estimating the state vector $\underline{s}_p(t)$ and the associated quadratic term $\underline{s}_p(t)\underline{s}_p^T(t)$ given the input data vector $\underline{z}$ and assuming the parameter vector $\underline{\theta}$ is known (from the previous iteration). These estimates can be computed using a Kalman smoother [3].

To modify the block algorithm above to obtain a sequential/adaptive algorithm, we use a causal Kalman filter whose parameters are continuously updated rather than a Kalman smoother to estimate $\underline{s}_p(t)$ and $\underline{s}_p(t)\underline{s}_p^T(t)$ based on the data up to time "t" and the parameter estimate at time "$t-1$". In addition at time "t" we need to compute the parameter estimate $\underline{\theta}(t)$. The most straightforward modification of equations (9)-(11) is to let the block length increase with "t". This corresponds in (9)-(11), to replacing the iteration index "l" by the time index "t" and replacing the sample averages $\frac{1}{N}\sum_{t=1}^{N}(.)$ by $\frac{1-\gamma}{1-\gamma^t}\sum_{\tau=1}^{t}(.)\gamma^{t-\tau}$, where γ is an exponential "forgetting" factor whose value is between 0 and 1. The resulting algorithm weighs the current data more heavily than the past data, and also allows a recursive procedure for updating the parameters. The resulting parameter update equations are:

$$\underline{\alpha}(t+1) = -\left[\sum_{\tau=1}^{t} E^{(\tau)}\{\underline{s}_{p-1}(\tau-1)\underline{s}_{p-1}^T(\tau-1)\}\gamma^{t-\tau}\right]^{-1} \times \sum_{\tau=1}^{t} E^{(\tau)}\{\underline{s}_{p-1}(\tau-1)s(t)\}\gamma^{t-\tau} \qquad (17)$$

$$\sigma_s^2(t+1) = \frac{1-\gamma}{1-\gamma^t}\left\{\sum_{\tau=1}^{t} E^{(\tau)}\{s(\tau)^2\}\gamma^{t-\tau} + \underline{\alpha}^T(t)\sum_{\tau=1}^{t} E^{(\tau)}\{\underline{s}_{p-1}(\tau-1)s(\tau)\}\gamma^{t-\tau}\right\} \qquad (18)$$

$$\sigma_\epsilon^2(t+1) = \frac{1-\gamma}{1-\gamma^t}\sum_{\tau=1}^{t}\left[z^2(\tau) - 2z(\tau)E^{(\tau)}\{s(\tau)\} + E^{(\tau)}\{s^2(\tau)\}\right]\gamma^{t-\tau}, \qquad (19)$$

where $E^{(t)}\{(.)\} \equiv E\{(.)|z(1),\ldots,z(t)\}$. All the necessary conditional expectations are computed using a Kalman filter whose parameters are continuously updated.

The cancelling signal $c(t+M)$ is obtained by estimating $s(t+M)$ based on $E^{(t)}\{\underline{s}_p(t)\}$ and $\underline{\theta}(t)$, the estimates of $\underline{s}_p(t)$ and $\underline{\theta}$ at time "t". It is straightforward to show that since $u(t)$ is white,

$$\begin{aligned}\hat{\underline{s}}_p(t+M|t) &\equiv E\{\underline{s}_p(t+M)|z(1),\ldots,z(t);\underline{\theta}(t)\} \\ &= \Phi^M E^{(t)}\{\underline{s}_p(t)\}.\end{aligned}$$

The cancelling signal $c(t+M)$ is then taken as the negative of the last element of $\hat{\underline{s}}_p(t+M|t)$.

3.1 Gradient-Based Algorithm

An alternative to the EM algorithm is the use of the gradient-search algorithm for maximizing $\log f_{\underline{z}}(\underline{z};\underline{\theta})$:

$$\underline{\theta}^{(l+1)} = \underline{\theta}^{(l)} + \delta^{(l)}.\frac{1}{N}.\frac{\partial \log f_{\underline{z}}(\underline{z};\underline{\theta})}{\partial\underline{\theta}}\Big|_{\underline{\theta}=\underline{\theta}^{(l)}} \qquad (20)$$

where $\underline{\theta}^{(l)}$ is the estimate of $\underline{\theta}$ after l iteration cycles, $\delta^{(l)}$ is the step-size on the l-th iteration cycle (this step-size can be different for different components of $\underline{\theta}$), and N corresponds to the number of data samples. For sufficiently small step-size, this algorithm converges to a stationary point of the log-likelihood function.

The partial derivatives $\frac{\partial}{\partial\underline{\theta}}\log f_{\underline{z}}(\underline{z};\underline{\theta})|_{\underline{\theta}=\underline{\theta}^{(l)}}$, where $\underline{z}$ denotes the observed (incomplete) data, can be calculated by taking the conditional expectation of the derivatives $\frac{\partial}{\partial\underline{\theta}}\log f_{\underline{y}}(\underline{y};\underline{\theta})$, where y denotes the complete data, i.e,

$$\frac{\partial}{\partial\underline{\theta}}\log f_{\underline{z}}(\underline{z};\underline{\theta})|_{\underline{\theta}=\underline{\theta}^{(l)}} = E\left\{\frac{\partial}{\partial\underline{\theta}}\log f_{\underline{y}}(\underline{y};\underline{\theta})|\underline{z};\underline{\theta}^{(l)}\right\}\Big|_{\underline{\theta}=\underline{\theta}^{(l)}}. \qquad (21)$$

This identity was first presented in [6]. The conditional expected values in (21) can again be computed using a Kalman smoother whose parameters are based on $\underline{\theta}^{(l)}$. Using (21) to calculate the partial derivatives in (20) and following steps very similar to those taken in the previous section, we obtain the following sequential/adaptive parameter update equations [3]:

$$\underline{\alpha}(t+1) = \underline{\alpha}(t) - \delta(t).\frac{1}{\sigma_s^2(t)}.\frac{1-\gamma}{1-\gamma^t}\sum_{\tau=1}^{t}\left\{\left[E^{(\tau)}\{\underline{s}_{p-1}(\tau-1)s(\tau)\} + E^{(\tau)}\{\underline{s}_{p-1}(\tau-1)\underline{s}_{p-1}^T(\tau-1)\}\underline{\alpha}(\tau)\right]\gamma^{t-\tau}\right\}$$

$$\sigma_s^2(t+1) = \sigma_s^2(t) + \delta(t)\left\{-\frac{1}{2\sigma_s^2(t)} + \frac{1}{2[\sigma_s^2(t)]^2}.\frac{1-\gamma}{1-\gamma^t}\sum_{\tau=1}^{t}\left\{\left[E^{(\tau)}\{s^2(\tau)\} + 2\underline{\alpha}^T(t)E^{(\tau)}\{\underline{s}_{p-1}(\tau-1)s(\tau)\} + \underline{\alpha}^T(\tau)E^{(\tau)}\{\underline{s}_{p-1}(\tau-1)\underline{s}_{p-1}^T(\tau-1)\}\underline{\alpha}(\tau)\right]\gamma^{t-\tau}\right\}\right\}$$

$$\sigma_\epsilon^2(t+1) = \sigma_\epsilon^2(t) + \delta(t).\left\{-\frac{1}{2\sigma_\epsilon^2(t)} + \frac{1}{2[\sigma_\epsilon^2(t)]^2}.\frac{1-\gamma^t}{1-\gamma}\sum_{\tau=1}^{t}\left\{\left[z^2(\tau) - 2z(\tau)E^{(\tau)}\{s(\tau)\} + E^{(\tau)}\{s^2(\tau)\}\right]\gamma^{t-\tau}\right\}\right\}$$

The obvious advantage of the Gradient-Based algorithm over the algorithm of (17)-(19) is that this alfunction $H(z)$. The signal $r(t)$ is the signal generated by the processor and corresponds to the input to the cancelling speaker. The block labeled Z^{-M} represents the delay from the speaker output to the microphone with $c(t)$ denoting the cancelling signal at the microphone input. The noise source $\sigma_\epsilon v(t)$ represents the measurement error in the microphone and is assumed to be uncorrelated with $\sigma_s u(t)$.

3 ADAPTIVE NOISE CANCELLATION ALGORITHM

The EM algorithm is an iterative procedure for maximum likelihood estimation using the concept of complete and incomplete data [2,3]. We shall first develop this algorithm in the context of our noise cancellation problem as a time-domain block iterative algorithm. This block algorithm is then modified to obtain an adaptive/sequential algorithm.

As indicated in figure 2 $z(t)$ is assumed to be of the form

$$z(t) = s(t) + \sigma_\epsilon v(t) \quad (1)$$

where $v(t)$ is zero-mean unit variance Gaussian noise. Furthermore $s(t)$ is modeled as the output of an all-pole(autoregressive) system, i.e

$$s(t) = -\sum_{k=1}^{p} \alpha_k s(t-k) + \sigma_s u(t) \quad (2)$$

with $u(t)$ a zero-mean unit variance white Gaussian process, independent of $v(t)$. We define the vector of unknown parameters, as

$$\underline{\theta} = [\underline{\alpha}^T, \sigma_s^2, \sigma_\epsilon^2]^T \quad (3)$$

where $\underline{\alpha}$ is the vector of autoregressive parameters. We further define the observed data vector $\underline{z}$, signal vector $\underline{s}$, and state vector $\underline{s}_p(t)$ as

$$\begin{aligned} \underline{z} &= [z(1), z(2), \ldots, z(N)]^T && (4)\\ \underline{s} &= [s(-p+1), s(-p+2), \ldots, s(N)]^T && (5)\\ \underline{s}_p(t) &= [s(t-p), s(t-p+1), \ldots, s(t)]^T. && (6) \end{aligned}$$

In formulating the EM algorithm for our problem, the vector $\underline{z}$ represents the incomplete data. The complete data is then taken to be the vector $\underline{y}^T = [\underline{z}^T; \underline{s}^T]$.

On each iteration of the EM algorithm, the parameter vector is re-estimated by maximizing the expectation of the log-likelihood function of the complete data, with the expectation conditioned on knowing the incomplete data and using the parameter vector estimate obtained on the previous iteration, i.e

$$\underline{\theta}^{(l+1)} = \arg\max_{\theta} \; E\{\log f_{\underline{y}}(\underline{y}; \underline{\theta}) | \underline{z}; \underline{\theta}^{(l)}\}. \quad (7)$$

Using the assumption that $s(t)$ is Gaussian and satisfies equation (2) , it is straightforward to show that

$$\begin{aligned} E\{\log f_{\underline{y}}(\underline{y}; \theta) | \underline{z}; \theta^{(l)}\} &= \log f(\underline{s}_{p-1}(0); \theta) - N \log \sigma_s \\ &- \tfrac{1}{2\sigma_s^2} \textstyle\sum_{t=1}^{N} \left[E^{(l)}\{s(t)^2\} + 2\underline{\alpha}^T E^{(l)}\{\underline{s}_{p-1}(t-1)s(t)\}\right. \\ &+ \left.\underline{\alpha} E^{(l)}\{\underline{s}_{p-1}^T(t-1)\underline{s}_{p-1}(t-1)\}\underline{\alpha}\right] \\ &- \tfrac{1}{2\sigma_\epsilon^2} \textstyle\sum_{k=1}^{N} \left[z(t)^2 - 2z(t)E^{(l)}\{s(t)\} + E^{(l)}\{s(t)^2\}\right] \\ &- N \log \sigma_\epsilon \end{aligned} \quad (8)$$

where $E^{(l)}$ denotes the conditional expected value , conditioned on knowing $\underline{z}$, and using the parameter vector obtained on the l-th iteration. The parameter vector estimate for the $(l+1)$-st iteration, $\underline{\theta}^{(l+1)}$, is then obtained by maximizing $E^{(l)}\{\log f_{\underline{y}}(\underline{y}; \theta)\}$. Specifically,

$$\underline{\alpha}^{(l+1)} = -\left[\sum_{t=1}^{N} E^{(l)}\{\underline{s}_{p-1}(t-1)\underline{s}_{p-1}^T(t-1)\}\right]^{-1} \times \sum_{t=1}^{N} E^{(l)}\{\underline{s}_{p-1}(t-1)s(t)\} \quad (9)$$

$$(\sigma_s^2)^{(l+1)} = \frac{1}{N}\sum_{t=1}^{N} E^{(l)}\{s(t)^2\} + (\underline{\alpha}^l)^T \sum_{t=1}^{N} E^{(l)}\{\underline{s}_{p-1}(t-1)s(t)\} \quad (10)$$

$$(\sigma_\epsilon^2)^{(l+1)} = \frac{1}{N}\sum_{t=1}^{N} \left[z^2(t) - 2z(t)E^{(l)}\{s(t)\} + E^{(l)}\{s^2(t)\}\right]. \quad (11)$$

The expectations required in equations (9)-(11) can be computed using a Kalman smoother whose parameters are based on the estimate of $\underline{\theta}$ obtained on the l-th iteration. To establish the Kalman filter structure, we express equations (1) and (2) in state space form as

$$\begin{aligned} \underline{s}_p(t) &= \Phi \underline{s}_p(t-1) + \underline{g}u(t) && (12)\\ z(t) &= \underline{h}^T \underline{s}_p(t) + \sigma_\epsilon v(t) && (13) \end{aligned}$$

where Φ is the $(p+1) \times (p+1)$ matrix

$$\Phi = \begin{bmatrix} 0 & I \\ 0 & -\underline{\alpha}^T \end{bmatrix} \quad (14)$$

$\underline{h}^T$ is the $1 \times (p+1)$ vector

$$\underline{h}^T = [0, \ldots, 0, 1] \quad (15)$$

and $\underline{g}$ is the $(p+1) \times 1$ vector

$$\underline{g}^T = [0, \ldots, 0, \sigma_s]. \quad (16)$$

gorithm does not require a matrix inversion on every time-step.

4 REAL TIME SIMULATIONS

The gradient-based sequential/adaptive algorithm was used to cancel the noise generated by three types of aircraft in real-time simulations. The algorithm was implemented on an AT&T DSP32C chip operating at 32KHz sampling rate. The order of the underlying AR process was assumed to be five, and σ_ϵ was fixed at 5% of the standard deviation of the unwanted noise. The required prediction time is dependent on the spacing between the cancelling speaker and the microphone. In our simulations, one step prediction corresponds to a spacing of 1.0cm and a prediction time of 31.25μsec. Figure 3 is a plot of the average noise attenuation versus the prediction time.

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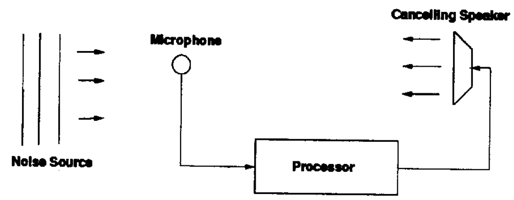

Figure 1: Generic single microphone ANC system

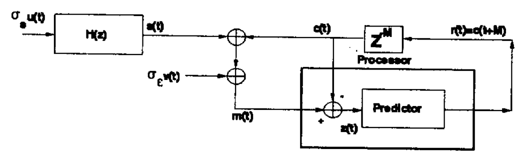

Figure 2: Idealized single microphone ANC model

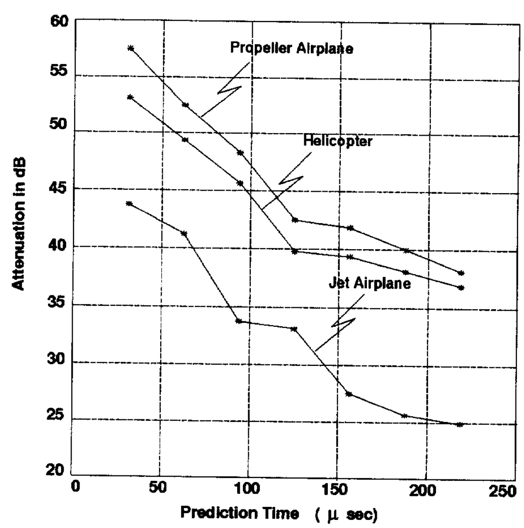

Figure 3: Attenuation obtained using our ANC algorithm. The measured attenuation values are indicated and are connected by straight lines.