Single-Sensor Active Noise Cancellation

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***Abstract*— Active noise cancellation is an approach to noise reduction in which a secondary noise source that destructively interferes with the unwanted noise is introduced. In general, active noise cancellation systems rely on multiple sensors to measure the unwanted noise field and the effect of the cancellation. This paper develops an approach that utilizes a single sensor. The noise field is modeled as a stochastic process, and a time-adaptive algorithm is used to adaptively estimate the parameters of the process. Based on these parameter estimates, a canceling signal is generated. In general, the transfer function characteristics from the canceling source to the error sensor need to be accounted for. If these can be accurately measured in advance and are invertible except for the propagation delay between the source and sensor, then the essential problem becomes one of predicting future values of the noise field. The algorithm developed is evaluated with both artificially generated noise and with recordings of aircraft noise.**

I. Introduction

Unwanted acoustic noise is a by-product of many industrial processes and systems. With active noise cancellation (ANC), a secondary noise source is introduced to generate an acoustic field that interferes destructively with the unwanted noise and thereby attenuates it [2], [3], [11].

Conventional ANC systems typically utilize several sensors: at least one to measure the noise field and a separate sensor to measure the canceled or attenuated noise. A conventional two-sensor ANC system consists of an input sensor, adaptive filter, canceling source, and error sensor, as depicted in Fig. 1. The input sensor is used to measure the unwanted noise at a location away from the error sensor and provides the input to the adaptive filter. In applications such as noise cancellation in a duct for which the noise propagation is essentially unidirectional, the input sensor is positioned upstream of the location at which the noise is to be canceled so that its output in effect anticipates or predicts the noise field at the location of the error sensor. The error sensor measures the residual acoustic field that is used to adjust the adaptive filter coefficients. Two-sensor systems are particularly effective when the input sensor can anticipate the noise field at the error sensor so that processing delay in the filter and any propagation delay between the canceling source and the error sensor are easily compensated for. In addition to the issues associated with deploying multiple sensors, a common difficulty with multiple sensor ANC systems is that there is typically some feedback from the canceling speaker to the input microphone. Several approaches have been proposed for dealing with this problem either by utilizing a configuration of canceling sources that minimizes the feedback or by taking the effect of feedback into account in the design of the adaptive filter [3].

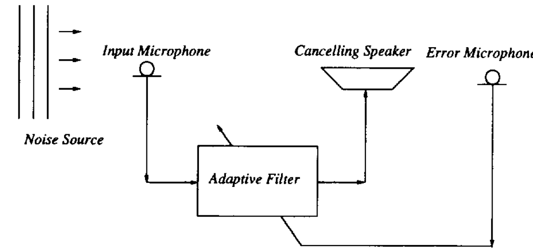

Fig. 1. Conventional active noise cancellation system.

In this paper, we present a new ANC system utilizing only a single sensor. As with conventional systems, a canceling source is used to generate a second acoustic field. However, in the system we develop, a single sensor is used to provide an estimate of both the original noise field and the canceled noise field.

To compensate for the propagation delay between the canceling speaker and the sensor, predicted values of the noise field are used. The prediction is based on modeling the noise field as a stochastic autoregressive process whose parameters are adaptively estimated. Because the parameter estimation is adaptive, the resulting ANC system can be used to cancel stationary as well as nonstationary noise.

In a recent paper by Zangi [13], the performance of a single sensor ANC algorithm is compared with that of the two sensor algorithm proposed by Burgess [1]. The results in this paper suggest that the noise attenuation levels obtained by the single sensor algorithm are typically 10–15 dB higher than the ones obtained by the two sensor algorithm. Note that in the two sensor algorithm, the canceling signal is derived from the output of the input sensor, whereas in the signal sensor algorithm, the canceling single is derived from the output of the error sensor. It is argued in [13] that the output of the error sensor carries much more information about the future values

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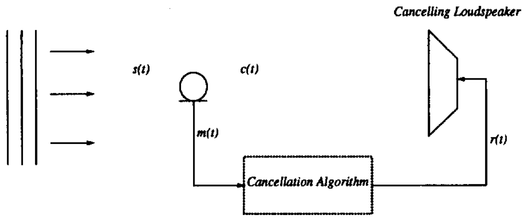

Fig. 2. Generetic single-microphone active noise cancellation system.

of the unwanted noise at the error sensor than the output of the input sensor located away from the error sensor. This is particularly true whenever the noise field is omnidirectional so that the output of the input sensor can no longer anticipate the noise at the error sensor.

Using the algorithms developed in this paper, a number of experiments were performed, both with artificially generated noise and with recordings of aircraft noise. With artificially generated noise, the performance of the proposed ANC system in a mean square sense was found to be close to that of the system that would exploit the exact noise characteristics rather than adaptively estimate them. Furthermore, in the case of nonstationary noise, the proposed system is able to adapt to the changing noise statistics.

In the context of aircraft noise, the algorithms were evaluated in a simulated environment to cancel the noise generated by a helicopter, a propeller airplane, and a jet airplane. Assuming that the canceling speaker and the microphone were three centimeters apart and that the transfer function between the canceling speaker and the microphone is a pure delay, the algorithm is able to attenuate the overall noise power by 45, 40, and 35 dB, respectively.

In Section II, we present our model for the cancellation environment and the unwanted noise. In Section III, we derive the ANC algorithm based on this model. Section IV discusses the performance of the algorithm on recorded aircraft noise.

II. Model Specification

A generic single-sensor ANC system is depicted in Fig. 2, where the microphone output is $m(t)$, and $r(t)$ is the input to the canceling loudspeaker. The microphone measures the sum of the unwanted noise $s(t)$ and the canceling signal $c(t)$. The objective is to generate $r(t)$ based on the measurements of $m(t)$ in such a way that the energy of $m(t)$ is minimized.

The block diagram for the single-sensor ANC system that we propose and develop in this paper is depicted in Fig. 3. The system $G(z)$ represents the overall transfer function from the canceling source input $r(t)$ to the sensor output $m(t)$ and incorporates the transfer functions of the source and the sensor together with the propagation delay between the source and sensor. The microphone output $m(t)$ is the sum of the unwanted noise $s(t)$, the canceling signal $c(t)$, and the measurement noise $v(t)$. The overall strategy is based on the observation that if $G(z)$ is known or can be adaptively estimated, then, since $r(t)$ is known exactly, an estimate of the uncancelled noise at the sensor can be obtained by subtracting out the component of the sensor output due to the canceling

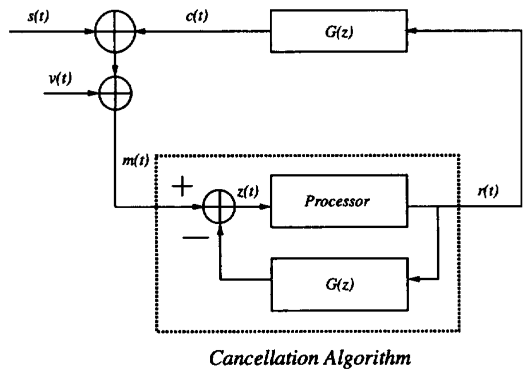

Fig. 3. Our single-microphone active noise cancellation system.

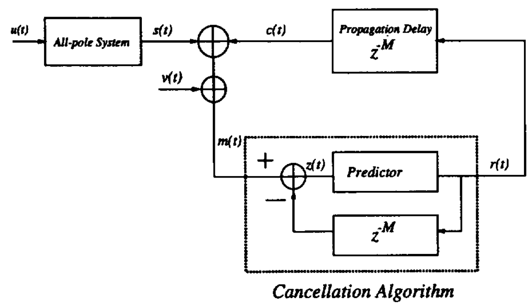

Fig. 4. Block diagram of our single-microphone active noise cancellation system.

source, i.e., we can extract $z(t) = s(t) + v(t)$ from the output of the microphone $m(t)$. The input to the canceling loudspeaker is then generated based on $z(t)$.

Although it is eventually important and of interest to fully develop the algorithm by adaptively estimating $G(z)$, we focus in this paper on the more idealized and simpler problem in which we only account for the propagation delay between the source and sensor. Since this delay is the result of the source and sensor separation, it is reasonable to assume that it is known and constant.

A block diagram representation for the overall system, with $G(z) = z^{-M}$, is shown in Fig. 4. In this figure, $s(t)$ is the unwanted noise at the microphone and is modeled as the output of an all-pole transfer function driven by white noise, i.e., $s(t)$ is modeled as an autoregressive (AR) process. $v(t)$ represents the measurement noise in the microphone.

Recall that the objective is to choose $r(t)$ such that the energy of the residual signal measured by the microphone $m(t)$ is minimized; furthermore, $r(t)$ must be generated based on $z(\tau) : \tau = 1, \ldots, t$. It is then easy to see that in the system of Fig. 4, the minimizing choice for $r(t)$ is given by the following conditional expectation:

$$r(t) = -E\{z(t+M)|z(t), z(t-1), \ldots, z(t)\} \quad (1)$$

where $z(t) = s(t) + v(t)$. Note that $z(t)$ is what the microphone would have measured, if the canceling signal was turned off. By choosing $r(t)$ according to (1), the output of the microphone $m(t)$ becomes equal to the following prediction error:

$$m(t) = z(t) - E\{z(t)|z(t-M), z(t-M-1), \ldots, z(1)\}. \quad (2)$$

Our approach is to estimate the parameters of the AR model through the adaptive algorithm derived in Section III. These parameters along with the measurements of the microphone are then used to predict $z(t)$. These predicted values are in turn used to obtain the input to the canceling loudspeaker according to (1).

Although, in this paper, we focus specifically on signal prediction and noise cancellation, we note that the algorithms presented here can also be used for the purpose of signal enhancement since they generate the signal and parameter estimates in the presence of noise. Since the algorithms are sequential/adaptive in nature, they may be particularly useful in the context of enhancing nonstationary signals such as speech in the presence of nonstationary noise.

III. Single Sensor Adaptive Algorithm

The model for the signal $z(t)$ is

$$z(t) = s(t) + v(t) \quad (3)$$

where

$$s(t) = -\sum_{k=1}^{p} \alpha_k s(t-k) + u(t) \quad (4)$$

where $u(t)$ and $v(t)$ are statistically independent zero mean white Gaussian processes with average powers of σ_u^2 and σ_v^2, respectively.

Given signal observations up to time t, the minimum mean square error (m.m.s.e.) estimate of $s(t+m)$ is given by the conditional expectation:

$$\hat{s}(t+m) = E\{s(t+m)|z(1), z(2), \ldots, z(t)\}. \quad (5)$$

Note that $E\{z(t+M)|z(t), \ldots, z(1)\} = \hat{s}(t+M)$ since $z(t+M) = s(t+M) + v(t+M)$ and $v(t+M)$ is independent of $z(1), \ldots, z(t)$. This implies that (1) can be rewritten as $r(t) = -\hat{s}(t+M)$. Hence, in the remainder of this section, we will concentrate on adaptively calculating $\hat{s}(t+M)$.

If we assume that the parameters $\alpha_1, \alpha_2, \ldots, \alpha_p, \sigma_u^2$, and σ_v^2 are precisely known, then this conditional expectation can be computed efficiently using the Kalman filtering equations. Toward this end, we represent (3) and (4) in state-space form as

$$\mathbf{x}(t) = \Phi \mathbf{x}(t-1) + \mathbf{e}_1 u(t) \quad (6)$$

$$z(t) = \mathbf{e}_1^T \mathbf{x}(t) + v(t) \quad (7)$$

where $\mathbf{x}(t)$ is the $(p+1) \times 1$ state vector defined by

$$\mathbf{x}(t) = [s(t)s(t-1)\cdots s(t-p)]^T \quad (8)$$

Φ is the $(p+1) \times (p+1)$ transition matrix

$$\Phi = \begin{bmatrix} -\alpha_1 & & -\alpha_2 & \cdots & \alpha_p & 0 \\ 1 & \ddots & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ 0 & & \cdots & 0 & 1 & 0 \end{bmatrix} \quad (9)$$

and $\mathbf{e}_1$ is the $(p+1) \times 1$ unit vector

$$\mathbf{e}_1 = [1 \quad 0 \quad \cdots \quad 0]^T. \quad (10)$$

Denote by

$$\hat{\mathbf{x}}(t|t) = E\{\mathbf{x}(t)|z(1), \ldots z(t)\} \quad (11)$$

the state estimate based on data to time t and by

$$\mathbf{P}(t|t) = E\{[\hat{\mathbf{x}}(t|t) - \mathbf{x}(t)][\hat{\mathbf{x}}(t|t) - \mathbf{x}(t)]^T|z(1), \ldots z(t)\} \quad (12)$$

the associated error covariance matrix.

Then, using the standard Kalman filter formulation, $\hat{\mathbf{x}}(t \mid t)$ and $P(t \mid t)$ can be computed sequentially in time, in two stages, as follows:[1]

Propagation Equations:

$$\hat{\mathbf{x}}(t|t-1) = \Phi \hat{\mathbf{x}}(t-1|t-1) \quad (13)$$

$$\mathbf{P}(t|t-1) = \Phi \mathbf{P}(t-1|t-1)\Phi^T + \sigma_u^2 \mathbf{e}_1 \mathbf{e}_1^T \quad (14)$$

Updating Equations:

$$\hat{\mathbf{x}}(t|t) = \hat{\mathbf{x}}(t|t-1) + \mathbf{k}(t)[z(T) - \mathbf{e}_1^T \hat{\mathbf{x}}(t|t-1)] \quad (15)$$

$$\mathbf{P}(t|t) = [I - \mathbf{k}(t)\mathbf{e}_1^T]\mathbf{P}(t|t-1) \quad (16)$$

where $\mathbf{k}(t)$ is the Kalman gain given by

$$\mathbf{k}(t) = \frac{1}{\mathbf{e}_1^T \mathbf{P}(t|t-1)\mathbf{e}_1 + \sigma_v^2} \mathbf{P}(t|t-1)\mathbf{e}_1. \quad (17)$$

The first component of $\hat{\mathbf{x}}(t \mid t)$ is the estimate of $s(t)$ based on data up to time t. To obtain the signal estimate at time $(t+m)$ as required by (5), we use (4) together with $\hat{\mathbf{x}}(t|t)$ to obtain the predicted values $\hat{s}(t+1|t), \hat{s}(t+2|t), \ldots \hat{s}(t+m|t)$, specifically

$$\hat{s}(t+\tau|t) = -\sum_{k=1}^{p} \alpha_k \hat{s}(t+\tau-k|t) \quad \tau = 1, 2, \ldots m. \quad (18)$$

The Kalman filtering equations require α, σ_u^2, and σ_v^2. Since these parameters are not available, they must be estimated as well.

The signal parameters satisfy the Yule-Walker equation

$$\mathbf{R} \begin{bmatrix} 1 \\ \\ \alpha \end{bmatrix} = \begin{bmatrix} \sigma_u^2 \\ \\ \mathbf{0} \end{bmatrix} \quad (19)$$

[1] These equations can be simplified by exploiting the structure of Φ and $\mathbf{e}_1$, as was done in [12]; however, this is not essential to our development here.

where $\mathbf{o}$ is the $p \times 1$ vector of zeros, α is the $p \times 1$ vector of the AR parameters

$$\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_p]^T, \tag{20}$$

and $\mathbf{R}$ is the $(p+1) \times (p+1)$ signal correlation matrix

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^T(t)\}, \tag{21}$$

where $\mathbf{x}(t)$ is the state vector defined in (8).

The estimate of the noise spectral level is given by

$$\sigma_v^2 = E\{[z(t) - s(t)]^2\}. \tag{22}$$

Denote by

$$\theta = \begin{bmatrix} \alpha \\ \sigma_u^2 \\ \sigma_v^2 \end{bmatrix} \tag{23}$$

the vector of unknown parameters and by

$$\hat{\theta}(t) = \begin{bmatrix} \hat{\alpha}(t) \\ \widehat{\sigma_u^2}(t) \\ \widehat{\sigma_v^2}(t) \end{bmatrix} \tag{24}$$

its estimate based on data up to time t.

Then, in accordance with (19), we consider generating the estimates of the signal parameters in such a way that the following equation is satisfied:

$$\widehat{\mathbf{R}}(t)\begin{bmatrix} 1 \\ \hat{\alpha}(t+1) \end{bmatrix} = \begin{bmatrix} \widehat{\alpha_u^2}(t+1) \\ \mathbf{0} \end{bmatrix} \tag{25}$$

where $\widehat{\mathbf{R}}(t)$ is the estimate of $\mathbf{R}$, which is obtained by performing the weighted averaging

$$\widehat{\mathbf{R}}(t) = \frac{1}{\sum_{\tau=1}^{t} \lambda^{t-\tau}} \sum_{\tau=1}^{t} \lambda^{t-\tau} \widehat{\mathbf{x}(\tau)\mathbf{x}^T}(\tau) \tag{26}$$

where

$$\widehat{\mathbf{x}(t)\mathbf{x}^T}(t) \overset{\Delta}{=} \hat{\mathbf{x}}(t|t)\hat{\mathbf{x}}^T(t|t) + \mathbf{P}(t|t) \tag{27}$$

and where $\hat{\mathbf{x}}(t|t)$ and $\mathbf{P}(t|t)$ are the estimate of the state and its covariance, which are computed using the Kalman filtering equations (13)–(16), where instead of θ, we use the current estimate $\hat{\phi}(t)$, i.e.

Propagation Equations:

$$\hat{\mathbf{x}}(t|t-1) = \hat{\mathbf{\Phi}}(t)\hat{\mathbf{x}}(t-1|t-1) \tag{28}$$

$$\mathbf{P}(t|t-1) = \hat{\mathbf{\Phi}}(t)\mathbf{P}(t-1|t-1)\hat{\mathbf{\Phi}}^T(t) + \widehat{\sigma_u^2}(t)\mathbf{e}_1\mathbf{e}_1^{\mathrm{T}} \tag{29}$$

Updating Equations:

$$\hat{\mathbf{x}}(t|t) = \hat{\mathbf{x}}(t|t-1) + \hat{\mathbf{k}}(t)\left[z(t) - \mathbf{e}_1^{\mathrm{T}}\hat{\mathbf{x}}(t|t-1)\right] \tag{30}$$

$$\mathbf{P}(t|t) = \left[I - \hat{\mathbf{k}}(t)\mathbf{e}_1^T\right]\mathbf{P}(t|t-1) \tag{31}$$

where $\hat{\Phi}(t)$ is the matrix defined in (9) computed at $\alpha = \hat{\alpha}(t)$, and $\hat{\mathbf{k}}(t)$ is the vector defined in (17) with σ_v^2 replaced by $\widehat{\sigma_v^2}(t)$.

Let $\hat{\mathbf{x}}(t|t)$ and $\widehat{\mathbf{x}(t)\mathbf{x}^T}(t)$ be partitioned as follows:

$$\hat{\mathbf{x}}(t|t) = \begin{bmatrix} \hat{s}(t|t) \\ \hat{\mathbf{s}}(t-1|t) \end{bmatrix}_{\updownarrow p},$$

$$\widehat{\mathbf{x}(t)\mathbf{x}^T}(t) = \begin{bmatrix} \widehat{s^2}(t) & \widehat{s(t)\mathbf{s}^T(t-1)} \\ \widehat{\mathbf{s}(t-1)s}(t) & \widehat{\mathbf{s}(t-1)\mathbf{s}^T}(t-1) \end{bmatrix}. \tag{32}$$

To obtain a sequential procedure for updating the signal parameter estimates, we define

$$\begin{matrix} 1 \updownarrow \\ p \updownarrow \end{matrix} \underbrace{\begin{bmatrix} Q_{11}(t) & \mathbf{Q}_{12}(t) \\ \mathbf{Q}_{21}(t) & \mathbf{Q}_{22}(t) \end{bmatrix}}_{1 \quad\quad p} = \mathbf{Q}(t) = \sum_{\tau=1}^{t} \lambda^{t-\tau} \widehat{\mathbf{x}(\tau)\mathbf{x}^T}(\tau) \tag{33}$$

$$= \widehat{\mathbf{x}(t)\mathbf{x}^{\mathrm{T}}}(t) + \lambda\mathbf{Q}(t-1). \tag{34}$$

Using (33) and (26), we express (25) as

$$\frac{1}{\sum_{\tau=1}^{t} \lambda^{t-\tau}} \begin{bmatrix} Q_{11}(t) & \mathbf{Q}_{12}(t) \\ \mathbf{Q}_{21}(t) & \mathbf{Q}_{22}(t) \end{bmatrix} \begin{bmatrix} 1 \\ \hat{\alpha}(t+1) \end{bmatrix} = \begin{bmatrix} \widehat{\sigma_u^2}(t+1) \\ \mathbf{0} \end{bmatrix} \tag{35}$$

which leads to the following update equations for the signal parameter estimates:

$$\hat{\alpha}(t+1) = -\mathbf{Q}_{22}^{-1}(t)\mathbf{Q}_{21}(t) \tag{36}$$

$$\widehat{\sigma_u^2}(t+1) = \frac{1-\lambda}{1-\lambda^t}[Q_{11}(t) + \mathbf{Q}_{12}(t)\hat{\alpha}(t)]. \tag{37}$$

where we have used the fact that

$$\frac{1}{\sum_{\tau=1}^{t} \lambda^{t-\tau}} = \frac{1-\lambda}{1-\lambda^t}.$$

Similarly, in accordance with (22), we generate the estimate of the noise power level using the following weighted averaging:

$$\widehat{\sigma_v^2} = \frac{1}{\sum_{\tau=1}^{t} \eta^{t-\tau}} \sum_{\tau=1}^{t} \eta^{t-\tau}[z^2(\tau) - 2z(\tau)\hat{s}(\tau|\tau) + \widehat{s^2}(\tau)]$$

$$= \frac{1-\eta}{1-\eta^t}\varsigma(t) \tag{38}$$

where $\varsigma(t)$ is computed recursively by

$$\varsigma(t) = \eta(\varsigma(t-1)) + [z^2(t) - 2z(t)\hat{s}(t|t) + \widehat{s^2}(t)]. \tag{39}$$

At each time step, the algorithm first estimates the current state (signal) using the latest parameter estimates in (28)–(31) and then updates the parameter estimates using the state estimate just computed and its covariance in (36)–(38).

The factors λ and η that appear in the cumulative averaging in (33) and (38), respectively, are numbers between 0 and 1. To maximize statistical stability, we choose $\lambda = 1$ and $\eta = 1$. Choosing λ and η to be strictly smaller than 1 corresponds to exponential weighting that gives more weight to current data samples and results in an adaptive algorithm that is capable of tracking nonstationary changes in the structure of the data.

As an alternative to the parameter update equations (33)–(39), we may consider a gradient-search algorithm for solving the Yule-Walker equation (19). In this case, instead of the signal correlation matrix $\mathbf{R}$, we use its estimate given by (26), and we proceed sequentially through the data using a stochastic gradient-type procedure. Replacing $\mathbf{R}$ in (19) by its current estimate $\widehat{\mathbf{R}}(t) = \frac{1-\lambda}{1-\lambda^t}\mathbf{Q}(t)$, where $\mathbf{Q}(t)$ is defined in (33), we obtain

$$\mathbf{Q}_{21}(t) + \mathbf{Q}_{22}(t)\alpha = \mathbf{0} \quad (40)$$

$$\sigma_u^2 - \frac{1-\lambda}{1-\lambda^t}[Q_{11}(t) + \mathbf{Q}_{12}(t)\alpha] = 0. \quad (41)$$

From (40) and (41) and using an approach similar to that used in compound decision problems (e.g., see [5]–[7], [9], [10]), the following sequential update equations for the signal parameters are suggested:

$$\hat{\alpha}(t+1) = \hat{\alpha}(t) - \gamma_t[\mathbf{Q}_{21}(t) + \mathbf{Q}_{22}(t)\hat{\alpha}(t)] \quad (42)$$

$$\widehat{\sigma_u^2}(t+1) = \widehat{\sigma_u^2}(t) - \gamma_t\Big[\widehat{\sigma_u^2}(t) - \frac{1-\lambda}{1-\lambda^t} \times [\mathbf{Q}_{21}(t) + \mathbf{Q}_{22}(t)\hat{\alpha}(t)]\Big] \quad (43)$$

where $\mathbf{Q}_{ij}(t)i,j = 1,2$ are computed recursively in t using (34).

Similarly, the noise power level is updated according to

$$\widehat{\sigma_v^2}(t+1) = \widehat{\sigma_v^2}(t) - \delta_t\left[\widehat{\sigma_v^2}(t) - \frac{1-\eta}{1-\eta^t}\varsigma(t)\right] \quad (44)$$

where $\varsigma(t)$ is computed recursively using (39) and where γ_t and δ_t are the step sizes being used. The advantage of using the algorithm in (42)–(44) is that it does not require matrix inversion, in contrast to the algorithm specified by (36)–(38), and therefore, it is computationally simpler. Although it may be possible to analyze these algorithms and to prove convergence under certain conditions, such an analysis is beyond the scope of this paper.

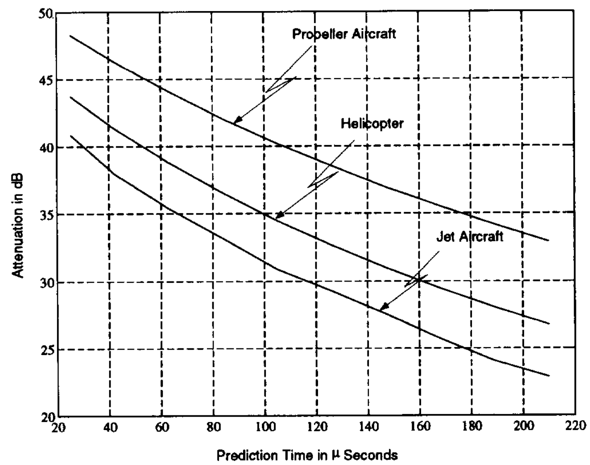

Fig. 5. Performance of the nongradient algorithm.

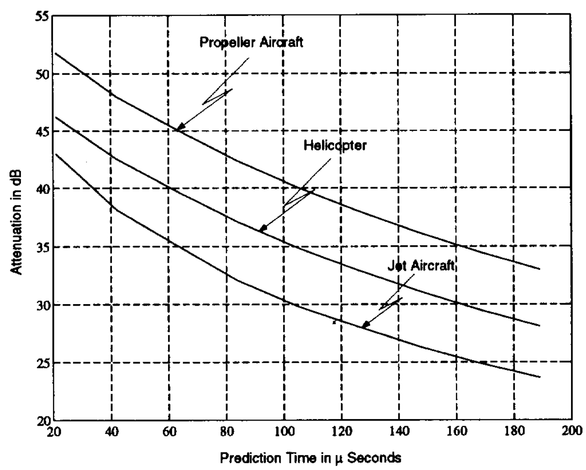

Fig. 6. Performance of the gradient algorithm.

IV. Algorithm Performance with Recorded Aircraft Noise

The results of applying both the gradient and the nongradient algorithm to three types of aircraft noise are presented in this section. These two algorithms were used on the noise generated by a propeller aircraft, a helicopter, and a jet aircraft. Recordings of these three types of noise were made from a microphone placed inside a set of headphones approximately 2 m away from each aircraft. Note that these recordings correspond to $z(t)$ in the system of Fig. 4.

Referring to Fig. 4, recall that the objective is to choose $r(t)$ so that the energy of the output of the microphone $m(t)$ is minimized. We see that choosing $r(t)$ according to (1) makes $m(t)$ equal to the prediction error in predicting $z(t)$ based on $z(\tau)$ $\tau = t - M, \ldots, 1$.

In our simulations, the order of the AR model for the unwanted noise was assumed to be five, and σ_v was fixed at 5% of the standard deviation of $z(t)$. The algorithms were tried with the order of the AR model ranging from three to nine, and it was found that the performance of the algorithm improves very little by increasing the order beyond five. The variance of the microphone measurement noise σ_u^2 can be estimated in the following way. The microphone is placed in a quiet environment with the ANC turned off so that the output of the microphone is $v(t)$. An estimate of σ_v^2 can then be obtained as

$$\sigma_v^2 \approx \frac{1}{N}\sum_{t=1}^{N} v^2(t). \quad (45)$$

The required prediction time in our simulations is determined by the spacing between the canceling speaker and the error microphone. In the simulations, one step prediction corresponds to a spacing of 0.625 cm between the canceling speaker and the error microphone and prediction time of 22.5 μs. Fig. 5 is a plot of the average noise attenuation versus the prediction time for the nongradient algorithm. Similarly, Fig. 6 is the plot of the average noise attenuation versus the prediction time for the gradient algorithm. Referring to Fig. 4, the attenuation is calculated as

$$\text{attenuation(dB)} = -10\log_{10}\frac{E\{m^2(t)\}}{E\{z^2(t)\}} \tag{46}$$

where $E\{z^2(t)\}$ is the average power for the original noise, and $E\{m^2(t)\}$ is the average power of the residual. Recall that the residual signal is equal to the prediction error.

It is interesting to note that the propeller noise is attenuated most, which is consistent with the fact that it has the longest correlation time of the three. Similarly, the jet noise is attenuated least since it has the shortest correlation time of all three. Furthermore, it is evident from these plots that the performance of the gradient algorithm is slightly better than the nongradient algorithm. Furthermore, gradient algorithm is then much less computationally intensive than the nongradient algorithm.

A number of other simulations were performed by applying the gradient and the nongradient algorithms to computer-generated autoregressive time series. Through these simulations, the performance of the two algorithms, in terms of mean square prediction error, was found to be very close to that of an ideal system, i.e., one that incorporates the true noise statistics.

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