A New Circuit for Communication Using Solitons

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ABSTRACT — **Solitons and the evolution equations that support them arise in the analysis of a wide range of physical phenomena including shallow water waves, piezoelectrics, and optical transmission in nonlinear fibers. Although such systems are nonlinear, they are exactly solvable and possess a class of solutions, known as solitons, which satisfy a form of superposition. This paper is an extension of a previous paper, in which a circuit implementation of the Toda lattice was examined for a variety of signal processing problems including multiple access communications, private or low power transmission, and multi-resolution transmission. In this paper, a novel diode ladder circuit is presented that more accurately implements the Toda lattice and is potentially more robust to additive channel corruption while providing more efficient modulation and demodulation.**

1 Introduction

Fourier representations of signals and their relationship to stationary processes and linear time-invariant (LTI) systems have had a tremendous impact on the design of many traditional communication systems. Recently, a class of nonlinear system models has been shown to possess many of the important properties of LTI systems. Although nonlinear, these systems are exactly solvable through a technique known as "inverse scattering" which can be viewed as an analog of the Fourier transform [1]. These systems admit a class of eigenfunctions, known as solitons, which satisfy a nonlinear form of superposition [3, 6].

In a recent paper [4], the use of solitons as modulating waveforms was discussed for a variety of signal processing and communication contexts. Specifically, soliton solutions of a nonlinear LC ladder circuit were considered for a multichannel AM and FM-like modulation, variable-rate pulse amplitude modulation with low-power applications, and priority or hierarchical multiple access channels. These represent only a few of the many ways in which solitons may be exploited for communications and signal processing.

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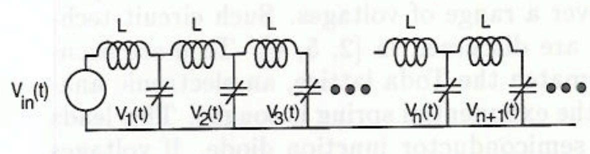

Figure 1: Nonlinear LC network of Hirota and Suzuki

2 Solitons in a Nonlinear Circuit

The communication schemes developed in [4] rely on the properties of the nonlinear transmission line model shown in Fig. 1. This LC circuit was first shown to support soliton solutions by Hirota and Suzuki [2] when the capacitor voltage v_n and charge q_n are related by

$$q(v_n) = C_0 V_b \ln(1 + v_n/V_b), \tag{1}$$

where V_b is the bias voltage and C_0 is a constant. In this case, the circuit is governed by the equations

$$\frac{d^2}{dt^2} \ln\left(1 + \frac{v_n}{V_b}\right) = \beta(v_{n-1} - 2v_n + v_{n+1}), \tag{2}$$

where $\beta = 1/LC_0V_b$, v_n is the voltage across the nth capacitor, $v_0 = v_{\text{in}}$, and L is the inductance. Suzuki, *et al.* have shown that this circuit is equivalent to the more familiar Toda lattice [6].

The Toda lattice describes a chain of masses (each of mass m) connected by nonlinear springs. If the displacement of the nth mass from its rest position is y_n, then the net displacement between the nth and $(n+1)$th masses is $r_n = y_{n+1} - y_n$. In the case that the springs obey a force relationship given by

$$f(r_n) = a\left(e^{-br_n} - 1\right), \qquad (3)$$

where a and b are arbitrary positive constants, the resulting system is called the Toda lattice.

The governing equations of motion for the lattice are given by

$$m\frac{d^2y_n}{dt^2} = a\left(e^{-b(y_n-y_{n-1})} - e^{-b(y_{n+1}-y_n)}\right). \quad (4)$$

An equivalent expression in terms of the forces on the springs, $f_n = f(r_n)$, is given by

$$\frac{d^2}{dt^2}\ln\left(1+\frac{f_n}{a}\right) = \frac{b}{m}(f_{n-1} - 2f_n + f_{n+1}), \quad (5)$$

which are equivalent to the equations in (2).

Nonlinear LC ladder realizations of such circuits have been constructed using biased varactor diodes [2, 5] to approximate the nonlinear capacitors over a range of voltages. Such circuit techniques are discussed in [2, 5, 4]. To more accurately match the Toda lattice, an electronic analog of the exponential spring is sought. This leads to the semiconductor junction diode. If voltages

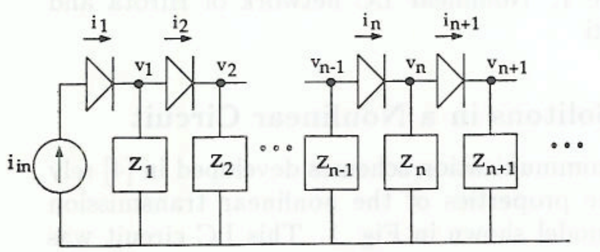

Figure 2: Diode ladder network.

v_{n-1} and v_n are applied to the terminals of a junction diode, the current through the device is well modeled by

$$i_n = I_s\left(e^{(v_{n-1}-v_n)/v_t} - 1\right), \qquad (6)$$

where I_s is the saturation current and v_t is the thermal voltage. If we place the diodes in a ladder configuration as shown in Fig. 2, then the current through the nth shunt impedance is given by

$$i_n - i_{n+1} = I_s\left(e^{(v_{n-1}-v_n)/v_t} - e^{(v_n-v_{n+1})/v_t}\right) \quad (7)$$

By analogy with (4), we see that if the shunt impedance is a "double capacitor" with a voltage-current relation given by

$$\frac{d^2v_n}{dt^2} = \alpha\hat{i}_n, \qquad (8)$$

where $\hat{i}_n = i_n - i_{n+1}$ is the current through the nth shunt impedance, then the governing equations become

$$\frac{d^2v_n}{dt^2} = \alpha I_s\left(e^{(v_{n-1}-v_n)/v_t} - e^{(v_n-v_{n+1})/v_t}\right), \quad (9)$$

and

$$\frac{d^2}{dt^2}\ln\left(1+\frac{i_n}{I_s}\right) = \frac{\alpha}{v_t}(i_{n-1} - 2i_n + i_{n+1}), \quad (10)$$

where $i_1 = i_{\text{in}} - i_2$. These are equivalent to the Toda lattice equations with $a/m = \alpha I_s$ and $b = 1/v_t$. A double capacitor can be realized using ideal operational amplifiers in the gyrator circuit shown in Fig. 2, which has the required impedance of $Z_n = \alpha/s^2 = R_3/R_1R_2C^2s^2$.

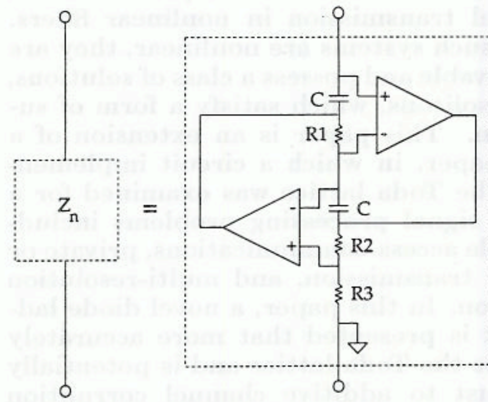

Figure 3: Double capacitor circuit design.

In addition to more closely matching the evolution equations for the Toda lattice, the diode ladder circuit can be more easily terminated than the nonlinear LC ladder. To find the proper termination, the diodes are replaced with their equivalent linearized resistance R_{eq} and the input impedance of the line is determined. This results in

$$Z_{\text{in}} = \frac{R_{\text{eq}}}{2} + \sqrt{\frac{R_{\text{eq}}^2}{4} + \frac{R_{\text{eq}}\alpha}{s^2}}. \qquad (11)$$

For the component values considered and for frequencies below 1 MHz, a load impedance consisting of a 10Ω resistor and a 60nF capacitor approximate 11 well and yield no almost reflections in practice.

When $i_{\text{in}}(t)$ in Fig. 2 is of the form

$$i_{\text{in}}(t) = I_s\Omega^2\text{sech}^2(\gamma t), \qquad (12)$$
$$\gamma = \Omega\sqrt{I_s\alpha/v_t},$$

it can be shown that (10) has the solution

$$i_n(t) = I_s\Omega^2\text{sech}^2(pn - \gamma t), \qquad (13)$$

where $\Omega = \sinh(p)$. This response corresponds to a single pulse traveling-wave solution, parameterized by the wavenumber, p, and is referred to as a soliton solution.

3 HSPICE Implementation

The diode lattice has been implemented with HSPICE using realistic component models for circuit simulations. The diode models used are d1n4449's with $I_s \approx 2$nA. To prevent saturation of the operational amplifiers in the double capacitor circuits, we have chosen to set $R_1 = R_2 = R_3 = 1\text{k}\Omega$. These values together with capacitors in the gyrators of 10nF permit soliton pulse widths of about 2μs with amplitudes of about 2mA. The double capacitors use precision LT1028A op-amps with a gain bandwidth product of about 65 MHz.

For the communication strategies of [4], two of the most important properties of the lattice solutions are illustrated in Fig. 4. First, these solu-

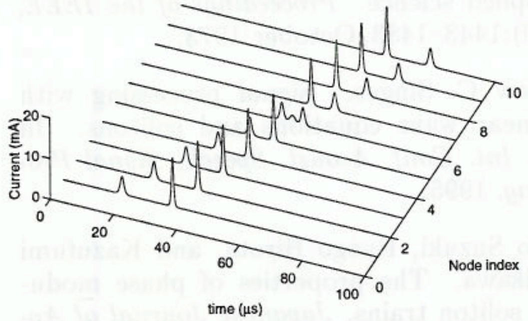

Figure 4: HSPICE simulation of two solitons in the diode lattice. Each horizontal trace shows the current through one of the diodes.

tions have amplitude-dependent pulse-width and velocity, yielding high-amplitude short-duration solitons which travel faster than those of lower-amplitude and longer-duration. Second, soliton solutions satisfy a form of nonlinear superposition whereby two solitons pass through one another leaving each virtually unchanged (except for a small phase shift) after their nonlinear interaction. A major difference between soliton solutions to this circuit and those of the nonlinear LC line lies in the scale of operation. Due to biasing constraints for the LC line, solitons were generally restricted to a small range wavenumbers, $p \approx 1$. Over this range, the traveling velocity of the solitons, which is proportional to $\sinh(p)/p$ does not vary greatly between solitons of different wavenumbers. In the new diode ladder circuit, however, since $I_s \approx 2$nA, we must have $p \approx 8$ for solitons to have amplitudes in the mA range. Due to the exponential nature of the sinh function, the velocities of solitons with slightly different currents in the mA range yield drastically different velocities. This enables modulation and demodulation circuits to separate messages into their component solitons with far fewer nodes. An interesting additional aspect of operating at higher soliton wavenumbers, as shown in Fig. 4, is the extra ripple that appears during the nonlinear interaction between the two solitons on node 6.

4 Noise Dynamics

For the nonlinear LC lattice, the effects of small amplitude white Gaussian noise on the dynamics of the soliton solutions was previously examined. It was found that for high signal-to-noise ratios the soliton solutions were relatively unaffected by the presence of the noise in the lattice. Furthermore, the noise dynamics were also qualitatively unaffected by the presence of the solitons. In the noise-only case, the small signal model for the lattice was a dispersive low pass filter. The effect of the solitons on the noise was to create a time varying gain in the lattice which traveled through the lattice with the solitons [4].

Since the range of wavenumbers over which the diode ladder circuit operates is drastically different from those of the nonlinear LC circuit, it is not surprising that the small signal analysis does not carry over. If the input to the lattice is composed of a soliton component and a noise component, $i_{\text{in}}(t) = S_{\text{in}}(t) + \tilde{i}_{\text{in}}(t)$, from [4], the non-soliton portion of the signal, $\tilde{i}_{\text{in}}(t)$ satisfies

$$\frac{d^2}{dt^2} \ln(1 + \frac{\tilde{i}_n}{(1+S_n)I_s}) = \frac{\alpha}{v_t}(\tilde{i}_{n-1} - 2\tilde{i}_n + \tilde{i}_{n+1}), \tag{14}$$

where S_n and $\tilde{i}_n$ are the components of the diode currents due to the soliton and non-soliton inputs respectively. In the nonlinear LC ladder analysis, we may assume that the contribution to the argument of the ln function from the noise component is small, and linearize the left hand side. In this case, since $I_s \approx 1$nA and S_n and $\tilde{i}_n$ are on the order of 1 mA, the noise term dominates the argument of the ln function. Since the soliton terms, S_n, are small compared to 1 and are very smooth, we may ignore their contribution to the left-hand side of (14). Therefore, the noise component of the response approximately satisfies (10), as it would if the soliton component were not present. Replacing the diodes with their linearized resistance, we see that the ladder becomes a very narrow lowpass filter, through which nearly all frequencies result in non-propagating (evanescent) waves. This result is confirmed empirically as shown in Fig. 5 for the two soliton example in white Gaussian noise with 20 dB signal to noise ratio. Note that although the noise is rapidly filtered out of the response, the velocity of the small amplitude soliton is slightly perturbed, resulting in the two solitons meeting a node later than in the noise-free case shown in Fig. 4. The apparent independence between the noise component and the soliton components and between the soliton components of different wavelengths is a result of a more fundamental property of the Toda lattice and relates to the inverse scattering method of solution whereby a large class of solutions to the Toda lattice comprise a discrete set of soliton components and a continuum of non-soliton components [6].

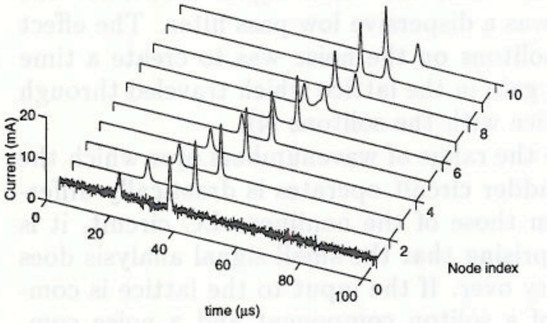

Figure 5: Solitons in the diode lattice at an SNR of 20 dB

5 Conclusions

In a previous paper [4], the use of a nonlinear LC ladder implementation of the Toda lattice and its soliton solutions was discussed for signal synthesis in a variety of modulation techniques. The ideas behind such strategies involve exploiting the independent behavior of solitons of different amplitudes as well as an apparent robustness to additive channel noise.

In this paper, a new circuit has been presented for soliton modulation that more accurately models the Toda lattice. In addition, this circuit can be more effectively terminated to avoid reflections. The diode ladder circuit also operates over a different range of soliton wavenumbers, allowing greater variation in the velocities of transmitted solitons, thus enabling more efficient modulators and demodulators in terms of the required number of nodes. Some of the implications of the new circuit on the noise analyses given in [4] were also discussed. The inverse scattering approach to the solution of the Toda lattice equations may provide insight into the higher order effects of additive corruption on the demodulation performance. This is an area of ongoing research.

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