SIGNALING TECHNIQUES USING SOLITONS

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ABSTRACT

Solitons and the nonlinear evolution equations that support them arise in the description of a wide range of nonlinear physical phenomena including shallow water waves, piezoelectrics, and optical transmission in nonlinear fibers. Although such systems are nonlinear, they are exactly solvable and possess a class of remarkably robust solutions, known as solitons, which satisfy a nonlinear form of superposition. By exploiting the properties of solitons, such nonlinear systems may be attractive for a variety of signal processing problems including multiple access communications, private or low power transmission, and multi-resolution transmission. We outline a number of modulation techniques using solitons, and explore some of the properties of such systems in the presence of additive channel corruption.

1. INTRODUCTION

Many classical signal processing algorithms rely heavily on the use of linear time-invariant (LTI) models and a Fourier representation of signals. Recently, a class of nonlinear system models has been shown to possess many of the important properties of LTI systems. Although nonlinear, these systems are exactly solvable through a technique known as "inverse scattering" which can be viewed as a nonlinear analog of the Fourier transform[1]. These systems also admit a class of eigenfunctions, known as solitons, which satisfy a form of nonlinear superposition[4, 7].

Solitons and their associated mathematics arise in the description of many physical phenomena and have recently been used by the telecommunications industry for long-distance transmission over nonlinear optical fibers[2]. The purpose of this paper is to consider exploiting the properties of solitons in a totally different context. We will address the use of nonlinear wave equations that support solitary wave solutions, or solitons, as a means of signal synthesis in several compelling communications scenarios. Specifically, we consider using solitons in a multichannel AM and FM-like modulation, variable-rate pulse amplitude modulation, and priority or hierarchical multiple access channels. These represent only a few of the many ways in which solitons can be exploited for communications and signal processing.

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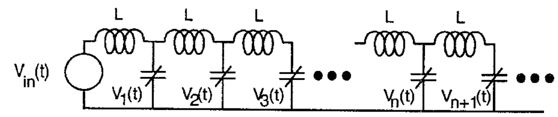

Figure 1: Nonlinear LC network of Hirota and Suzuki

2. SOLITONS IN A NONLINEAR CIRCUIT

The communication schemes developed in this paper rely on the properties of the nonlinear transmission line model shown in Fig. 1. This LC circuit was first shown to support soliton solutions by Hirota and Suzuki[3] when the capacitor voltage V and charge Q are related by

$$Q(V) = C_0 V_b \log(1 + V/V_b), \tag{1}$$

where V_b is the bias voltage and C_0 is a constant. In this case, the circuit is governed by the equations:

$$\frac{d^2}{dt^2}\log(1+\frac{V_n}{V_b}) = \frac{1}{LC_0V_b}(V_{n-1} - 2V_n + V_{n+1}), \tag{2}$$

where V_n is the voltage on the nth capacitor, $V_0 = V_{\text{in}}$, and L is the inductance. Suzuki *et al.* have shown that this circuit is equivalent to the more familiar Toda chain [7].

When $V_{\text{in}}(t)$ in Fig. 1 is of the form

$$V_{\text{in}}(t) = V_b\Omega^2\text{sech}^2(-\Omega\tau + \delta), \tag{3}$$

where $\tau = t/\sqrt{LC_0}$, it can be shown[3, 7] that (2) has the solution

$$V_n(t) = V_b\Omega^2\text{sech}^2(pn - \Omega\tau + \delta), \tag{4}$$

where $\Omega = \sinh(p)$. This response corresponds to a single-pulse traveling-wave solution, parameterized by the wave-number, p, and is referred to as a soliton solution.

For the purposes of this paper, two of the most important properties of these solutions are illustrated in Fig. 2. First, these solutions have amplitude dependent pulse-width and velocity, yielding tall narrow solitons which travel faster than short wide ones. Second, soliton solutions satisfy a form of nonlinear superposition whereby two solitons pass through one another leaving each virtually unchanged (except for a small phase shift) after their nonlinear interaction.

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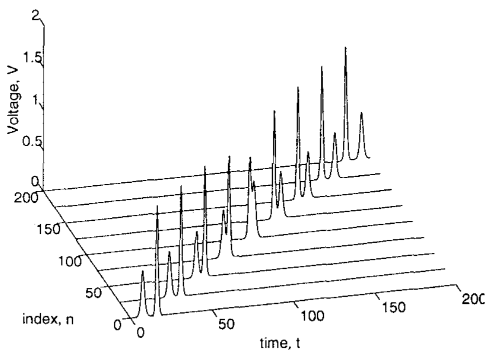

Figure 2: Two lattice solitons in the nonlinear network. Each horizontal trace shows the voltage across one of the capacitors.

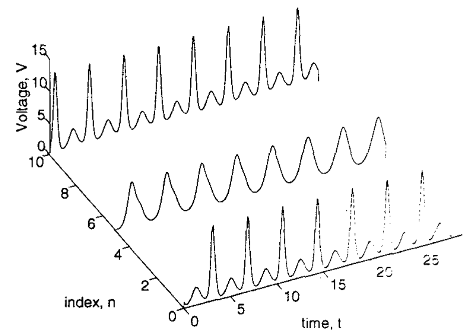

Figure 3: Soliton modulation of Suzuki *et al.*

3. COMMUNICATION WITH SOLITONS

Suzuki *et al.* proposed a method for secure communication in [6] using solitons as modulating waveforms and the nonlinear LC network of Fig. 1 for encoding and decoding the messages. Their basic signaling waveform was a periodic wave-train of solitons of two alternating wavenumbers. Since the two solitons have different amplitudes, when this signal is input to the nonlinear network, they will propagate with different velocities. At an appropriate point in the lattice, the faster soliton will "catch up" with the slower soliton from the previous period, resulting in their nonlinear superposition as seen in Fig. 3. The resulting sine-like waveform is transmitted as the encoded signal. At the receiver end, this wave-train is input into the nonlinear network, and again the fast solitons outpace the slow solitons resulting in their separation as seen in the voltage on a capacitor down the line. This process is illustrated in Fig. 3. The three traces correspond to the original soliton train consisting of two solitons, the single pulse train transmitted waveform, and the resulting separated two soliton train respectively. By slightly varying the relative phase between the soliton trains prior to their input into the network, an FM-like modulation can be achieved. Similarly, in [5], Suzuki *et al.* slightly modulate the amplitudes of the component soliton trains, realizing an AM-like transmission.

Although it is unclear whether or not these proposed techniques provide security, the underlying methodology suggests a more general framework for modulation of information by exploiting the properties of solitons. As a simple extension, the AM and FM techniques of Suzuki *et al.* can be generalized to include multiple solitons in each period and accommodate multiple channels of information, as shown in Fig. 4 for a four soliton example. In addition to issues regarding private communication, this modulation scheme may also provide additional spectral compression of the modulated waveforms.

Note that the same nonlinear network can support a spectrum of solitons ranging from those with small amplitudes and wide pulse-widths, to relatively large amplitudes and narrow pulse-widths. This indicates the potential for operating essentially the same modulator-demodulator networks at variable data rates depending on the bandwidth requirements. In the case of simple pulse amplitude modulation, where a bit is indicated by the presence or absence of a soliton, a tradeoff may be made between the data rates and the power in the transmitted signal. When communication requirements are low, small amplitude, wide solitons may be used and the necessary transmitted power is low. When bandwidth requirements increase, so does the requisite output power as narrower larger-amplitude solitons are used.

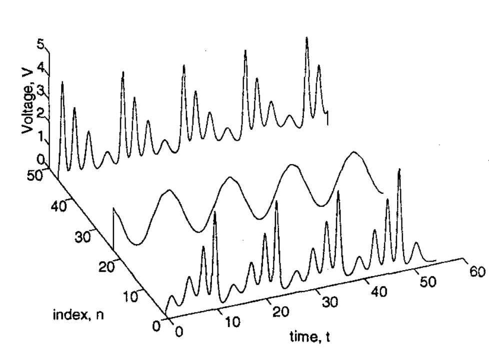

Figure 4: Four channel soliton multiplexing.

Such signaling techniques may also prove useful in the context of recent advances in multi-resolution signal representations. In a hierarchical modulation technique reminiscent of fractal modulation [8] a multi-resolution analysis of a signal may be transmitted such that each scale of resolution is encoded in a soliton of a different amplitude. If each of these soliton waveforms are combined and time-aligned according to the nonlinear superposition of the network, then the response of the receiver network to this signal as an input will be a gradual separation of each of the component soliton waveforms. In this manner, compactly represented signals might be constructed whereby varying amounts of processing, or equivalently, longer delay, yield signal representations of varying fidelity. As the received waveform is processed, the information present in the higher-amplitude, faster solitons emerges quickly from the bulk and may be decoded. As the message is passed further down the chain, the information in the next set of solitons may be decoded, and so on. This type of modulation may be useful in a variety of communications or broadcast contexts which contain a large number of receivers of variable processing power. This may also be useful in a signaling context with messages of variable priority.

4. NOISE DYNAMICS

Since the proposed communication strategies rely on using the nonlinear LC network to separate the encoded signal into component solitons, we need to address the effects additive channel corruption will have on the response of the network. Specifically, we will consider adding stationary white Gaussian noise to the input of the network $V_{\text{in}}(t)$ and characterize the resulting behavior of the system.

To begin, we look at the dynamics of small amplitude noise in the lattice in the absence of solitons. For V_{in} small, the circuit equations reduce to the linear LC ladder network given by

$$\frac{d^2V_n}{dt^2} = \frac{1}{LC_0}(V_{n-1} - 2V_n + V_{n+1}). \tag{5}$$

The response of the linear network to inputs of the form $V_{\text{in}}(t) = V_+ e^{j\omega t}$, is a traveling-wave solution of the form $V_n(t) = V_+ e^{j(\omega t - kn)}$, with a dispersion relation given by

$$k = 2\sin^{-1}(\omega/\omega_0), \qquad \omega_0 = \frac{2}{\sqrt{LC}}, \tag{6}$$

which has real solutions only for $\omega \leq \omega_0$. Frequencies above ω_0 correspond to evanescent (decaying) waves. The transfer function from the input voltage to the voltage at any node, n, corresponds to a dispersive low-pass filter, and is given by

$$H_N(j\omega) = \begin{cases} e^{-2j\sin^{-1}(\omega/\omega_0)n}, & |\omega| < \omega_0 \\ e^{(j\pi - 2\cosh^{-1}(\omega/\omega_0))n}, & \text{else.} \end{cases} \tag{7}$$

Therefore, for $n \gg 1$, the line has a magnitude characteristic approaching that of an ideal low-pass filter. Numerical simulations of the nonlinear line verify that the voltage at a capacitor far down the lattice will be low-pass in response to a small amplitude white Gaussian input. Although the autocorrelation of the noise at each capacitor is only effected by the magnitude response of (7), the cross-correlation between capacitors is also effected by the phase. The correlation between the voltages on capacitors m and n, is given by $R_{m,n}(\tau) = h_m(\tau) * h_n(-\tau) = h_m(\tau) * h_m(-\tau) * h_{n-m}(-\tau)$, where $h_m(\tau)$ is the inverse Fourier transform of $H_m(j\omega)$ in (7). Since $h_m(\tau) * h_m(-\tau)$ approaches the impulse response of an ideal low-pass filter for $m \gg 1$, we have

$$R_{m,n}(\tau) \approx \frac{\sin(\omega_0\tau)}{\pi\tau} * h_{n-m}(\tau). \tag{8}$$

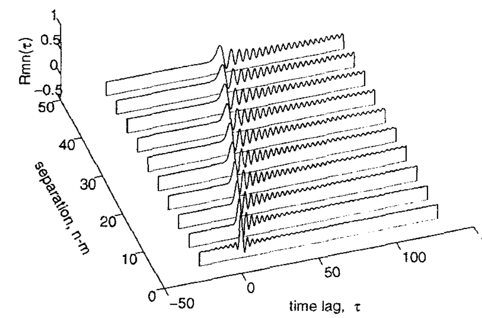

Figure 5: Cross correlation, $R_{m,n}(\tau)$, between the mth and the nth capacitor voltages in the linearized line.

In Fig. 5, $R_{m,n}(\tau)$ is shown for $n > m \gg 1$. Note that for ω small, $\sin^{-1}(\omega/\omega_0) \approx \omega/\omega_0$, and the line looks like a pure delay of $\alpha = 2(n-m)/\omega_0$. This would correspond to $R_{m,n}(\tau) = \sin(\omega_0(\tau - \alpha))/(\pi(\tau - \alpha))$. As shown in Fig. 5, for $n - m$ small, this approximation holds, but the dispersive nature of the line becomes apparent even for modest separations.

Returning to the nonlinear line, we observe that at high signal to noise ratios, the dynamics of the solitons are relatively unaffected, while the noise component remains low-pass. Shown in Fig. 6, is the voltage down the network in response to an input of two solitons embedded in white Gaussian noise at a signal to noise ratio of 20dB.

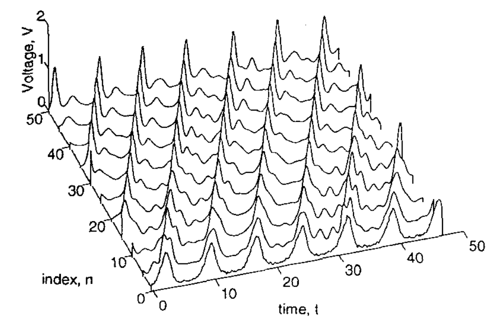

Figure 6: Two soliton example at 20dB SNR.

To characterize the behavior of the circuit when the received signal consists of solitons and noise, we employ a linearization approach. For high signal to noise ratios, we assume that the capacitor voltages are of the form, $V_n(t) = S_n(t) + v_n(t)$, when the input is of the form $V(t)_{\text{in}} = S_{\text{in}}(t) + v_{\text{in}}(t)$. Here $S_{\text{in}}(t)$ is the soliton component, and $v_{\text{in}}(t)$ is the noise component. We can rewrite (2) after factoring the argument of the logarithm and canceling terms from the known soliton solution, as

$$\frac{d^2}{dt^2}\log\left[\left(1+\frac{v_n}{(1+S_n)V_b}\right)\right]=\frac{1}{LC_0V_b}(v_{n-1}-2v_n+v_{n+1}), \quad (9)$$

which is an exact relation satisfied by the non-soliton portion of the response. Since v_n is small and the soliton solutions are smooth, we have

$$\frac{d^2v_n}{dt^2}\approx\frac{1+S_n}{LC_0}(v_{n-1}-2v_n+v_{n+1}). \quad (10)$$

Through this reasoning, the linear-time varying small signal model can be viewed over short time scales as a linear time-invariant chain, with a slowly varying value of C_0. This results in transfer functions that are low-pass filters with a time varying cutoff frequency equal to ω_0 when the soliton is far from the capacitor, and to $\omega_0\sqrt{1+S_n}$ as the soliton passes through. Thus, we would expect the variance of the voltage in each capacitor to rise from the nominal variance as a soliton passes through.

In order to verify this intuition, we linearize the exact dynamics about the known soliton trajectory, $S_n(t)$, and numerically integrate the corresponding Riccati equation for the capacitor voltage covariance. In Fig. 7, the resulting variance of the noise component on each capacitor is plotted as a function of time, and node index. Since the line was assumed initially at rest, there is an initial startup time, as well as an initial spatial transient at the beginning of the line, after which we see that the variance of the noise is amplified as each soliton passes through.

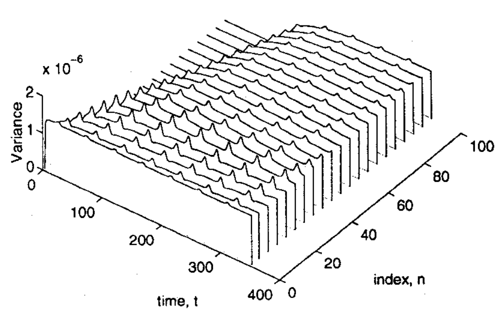

Figure 7: The variance of each capacitor voltage is shown as a function of time.

5. CONCLUSIONS AND FUTURE DIRECTIONS

We have discussed the use of a nonlinear LC network and its soliton solutions for signal synthesis in a variety of modulation techniques. The ideas behind such techniques involve exploiting the independent behavior of solitons of different amplitudes. In the AM/FM case, the different velocities of solitons were exploited to modulate and demodulate multiple signals. There may also be low-power transmission applications that exploit the tradeoff between amplitude and pulse-width (data rate), as well as multi-resolution signaling or priority messaging applications.

Before such systems can be considered for realistic communication problems, we have to understand how they operate in the presence of additive channel corruption. In this direction, we have explored some of the characteristics of how such systems respond to small amplitude Gaussian noise both in the presence and absence of solitons. We see that at high signal to noise ratios the soliton dynamics are relatively unaffected by the noise. However, the solitons have the effect of a time varying gain on the noise variance in the receiver. The stability of these and other soliton systems at moderate signal to noise ratios poses an interesting area for future research. As stated previously, a technique for exactly solving such systems involves a related linear inverse scattering problem. Exploring the implications of such modulation techniques on the linear scattering problem also represent an active area of ongoing research.

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