MONOCHROMATIC SINGLE-MODE EXCITATION IN SHALLOW WATER USING FEEDBACK CONTROL

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ABSTRACT

An algorithm is presented to excite a single mode in a shallow water channel using a vertical source array controlled by feedback from a reference hydrophone array. The algorithm iterates between computing the source weights based on its current estimate of the mode coupling in the channel, and updating its estimate of that coupling based on the modes observed at the feedback reference array. This allows us to excite high fidelity modes with confidence at a given location. The ability to control these modes depends on the accuracy with which they are observed. To this end, we compute the error for the linear least-squares mode estimator for scenarios where the feedback array does not span the entire water column. Finally, we present preliminary results obtained in a laboratory wave guide illustrating the successful convergence of the algorithm in a physical experiment.

1. INTRODUCTION

Normal modes are a convenient framework for examining acoustic propagation at mid-to-low frequencies in shallow water. The normal mode formulation allows the acoustic far field to be described using only a finite set of propagating modes. For a vertical array of sources in a perfectly known range-invariant environment, the complex weights of the sources can be chosen such that only one of these modes is propagating in the far field assuming the number of sources exceeds that of the propagating modes. Several laboratory experiments have pursued this goal in controlled environments.

Clay and Huang [1] and Gazanhes and Garnier [2] investigated single mode excitation in range-independent laboratory waveguides. Both experiments used open loop control schemes, where the source array weights were pre-computed based on detailed prior knowledge of a very simple propagation environment, then held constant for the entire experiment. Tindle *et al.* [3] examined single mode excitation and the induced coupling by propagation down a wedge. This experiment also used open loop control to compute the source array weights. This type of control is unfeasible in a realistic ocean environment as it relies on the modes propagating without coupling at least until the continuum energy has decayed. This assumption is not generally valid in an arbitrary shallow water environment. If the source array is deployed without detailed *a priori* knowledge of the environment, the open loop controller cannot guarantee it excited a single mode at any location, nor can it specify precisely how well it is performing even when it is working. This greatly limits the confidence of the inferences that can be made from observing the field further downrange. Lacking complete knowledge of the propagation environment, there is no way to be certain that inhomogeneities in the water column or sediment have not coupled either the desired excited mode or continuum energy into undesired modes before the continuum and near field contributions have completely attenuated.

Our solution to this problem is to move the location at which we are trying to control the field from the source array to the beginning of the far field. By placing a feedback hydrophone array at this location, the source array can be tuned using information received at this reference array so that only one mode is propagating into the far field. The field at this location has only a finite number of degrees of freedom, corresponding to the perfectly trapped modes propagating without loss of energy into the bottom, in contrast with the field at the source array, which consists of the modal continuum, a nearly infinite number of degrees of freedom. By changing from open loop control, which chooses its source array weights solely based on the environment at the source array, to closed loop control based on the discrete modes propagating at the feedback array, the degrees of freedom in the control problem are greatly reduced.

Section 2 of this paper briefly reviews the modal formulation of acoustic propagation, mainly to introduce the notation we will use. Section 3 describes our control algorithm in detail, including a number of numerical issues involved in the implementation of the algorithm. The es-

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timation error for the linear least-squares mode coefficient estimator as a function of array length is computed in Section 4. Finally, Section 5 presents preliminary results using the algorithm in a laboratory waveguide.

2. MODAL DESCRIPTION OF ACOUSTIC PROPAGATION

Time-harmonic pressure waves propagating in shallow water can be described by the equation

$$\rho(\mathbf{r})\nabla \cdot \left[\frac{1}{\rho(\mathbf{r})}\nabla p(\mathbf{r})\right] + k^2(\mathbf{r})p(\mathbf{r}) = 0, \qquad (1)$$

with appropriate boundary conditions [4], [5], [6]. In this equation, $\mathbf{r}$ is the location vector, $p(\mathbf{r})$ is the pressure and $\rho(\mathbf{r})$ is the density at that location, and $k(\mathbf{r})$ is the local wavenumber, $\omega/c(\mathbf{r})$, with ω denoting angular temporal frequency (rad/s), and $c(\mathbf{r})$ the sound speed at location $\mathbf{r}$. Assuming cylindrical symmetry, we can express the pressure in the far field as a weighted sum of discrete modes

$$p(r,z) = \sum_{m=1}^{M} d_m(r)\Psi_m(z;r), \qquad (2)$$

where $d_m(r)$ are the complex weights of the modes, or modal coefficients, at range r, and $\Psi_m(z;r)$ are the local vertical modes at range r. These local modes are the solutions to the vertical eigenfunction equation

$$\begin{aligned}\rho(z;r)\frac{d}{dz}\left(\frac{1}{\rho(z;r)}\frac{d\Psi_m(z;r)}{dz}\right) \quad + \quad \left(\frac{\omega}{c(r,z)}\right)^2 \Psi_m(z;r) \\ = \quad k_r(r)^2\Psi_m(z;r) \qquad (3)\end{aligned}$$

with the boundary conditions imposed by the environment at range r[6]. The variable $k_r(r)$ is the horizontal wavenumber associated with the mode. Because both $\Psi_m(z;r)$ and $d_m(r)$ are functions of range in general, both the shapes and weights of the modes can vary as they propagate outward in range. The variations in $d_m(r)$ due to range inhomogeneities are generally called mode coupling. The goal of our algorithm is to control the source array such that at some reference range $r = r_0$, all the coefficients $d_m(r_0) = 0$ except for that of the desired mode, $m = m_0$.

3. ALGORITHM DESCRIPTION

Our algorithm for generating a single mode uses a method of indirect feedback control from the adaptive control literature [7]. It alternates between a weight computation step and a channel estimate update step. In the weight computation step, the source array weights are calculated under the assumption that the current channel estimate is correct. The channel is excited using those source weights and the mode coefficients are estimated from the observed pressure field at the feedback array. Next, the channel estimate is updated based on the discrepancy between the modes observed and those expected based on the current channel estimate. The algorithm then returns to the weight computation step with this improved estimate of the channel, and repeats the process.

The algorithm assumes the vector of mode coefficients at the reference range $\mathbf{d}(r_0) = [d_1(r_0), \ldots, d_M(r_0)]^T$ is a linear function of the complex weights used at the source array. If the source array weights are written as a vector $\mathbf{w}$, the channel model is assumed to be

$$\mathbf{d}(r_0) = \mathbf{H}\mathbf{w}, \qquad (4)$$

where $\mathbf{H}$ is a transfer matrix incorporating all the effects of the mode shapes at the source array and the mode coupling between the source array and the feedback array. The desired $\mathbf{w}$ excites the channel such that $\mathbf{d}(r_0)$ is as close as possible in the least squares sense to a desired mode vector $\mathbf{d}_0$, which will be all zeros except for a 1 as the m_0 component.

The weight computation step finds the best $\widehat{\mathbf{w}}[n]$ in the least-squares sense, assuming the current channel estimate $\widehat{\mathbf{H}}[n-1]$ is correct. Precisely, this means

$$\widehat{\mathbf{w}}[n] = \arg\min_{\mathbf{w}} \|\mathbf{d}_0 - \widehat{\mathbf{H}}[n-1]\mathbf{w}\|^2. \qquad (5)$$

The solution to this minimization problem is

$$\widehat{\mathbf{w}}[n] = \left(\widehat{\mathbf{H}}[n-1]^H\widehat{\mathbf{H}}[n-1]\right)^{-1}\widehat{\mathbf{H}}[n-1]^H\mathbf{d}_0, \qquad (6)$$

where $(\cdot)^H$ denotes the Hermitian (conjugate transpose) operator.

To update $\widehat{\mathbf{H}}[n]$, the channel is excited with the source array using $\widehat{\mathbf{w}}[n]$ as the source array weights, and the pressure field $\mathbf{p}(r_0, \mathbf{z}_r)$ is observed, where $\mathbf{z}_r$ is a vector of the receiver depths at the reference array. These pressure observations are then filtered using the known mode shapes at the receiver array to obtain $\mathbf{d}[n]$, the estimate of the received modal coefficients for the n^{th} iteration[8]. $\widehat{\mathbf{H}}[n]$ is chosen to minimize the total error between the expected mode coefficients $\widehat{\mathbf{H}}[n]\widehat{\mathbf{w}}[\ell]$ and the observed coefficients $\mathbf{d}[\ell]$ for the interval $\ell = 1, \ldots, n$. Specifically, it satisfies the following criteria

$$\widehat{\mathbf{H}}[n] = \arg\min_{\mathbf{H}} \sum_{\ell=1}^{n} \gamma^{(n-\ell)}\|\mathbf{d}[\ell] - \mathbf{H}\widehat{\mathbf{w}}[\ell]\|^2, \qquad (7)$$

where γ is an exponential "forgetting factor" used to weight older data less than recent observations. This minimization is solved using a limited form of the recursive least squares (RLS) algorithm [9]. Specifically, the algorithm incorporates the additional information contained in new observations sequentially, but due to numerical issues does not exploit the matrix inversion lemma to update the inverse of the input auto-correlation matrix. The channel estimate is updated as follows

$$\widehat{\mathbf{H}}[n] = \gamma\widehat{\mathbf{H}}[n-1] + \mathbf{A}[n]\mathbf{k}[n]^H, \qquad (8)$$

where

$$\begin{aligned}\mathbf{A}[n] &= \mathbf{d}[n] - \widehat{\mathbf{H}}[n-1]\widehat{\mathbf{w}}[n] &(9)\\ \mathbf{k}[n] &= (\mathbf{\Phi}[n])^{-1}\widehat{\mathbf{w}}[n] &(10)\\ \mathbf{\Phi}[n] &= \sum_{k=1}^{n}\gamma^{n-k}\widehat{w}[k]\widehat{w}[k]^H &(11)\\ &= \gamma\mathbf{\Phi}[n-1] + \widehat{\mathbf{w}}[n]\widehat{\mathbf{w}}[n]^H. &(12)\end{aligned}$$

Intuitively, each mode, corresponding to a row of $\widehat{\mathbf{H}}[n-1]$, is corrected in the direction of $\mathbf{k}[n]^H$, and the size of the correction step is the appropriate element of the innovation vector $\mathbf{A}[n]$.

3.1. Numerical Issues

Most references on the RLS algorithm including [9] exploit the Matrix Inversion Lemma to minimize the computation at each iteration. We found that this formulation was too numerically sensitive for our application, and often produced spurious results when the algorithm ran for several hundred iterations. This behavior was due to extreme condition numbers of $\mathbf{\Phi}[n]$, since $\widehat{\mathbf{w}}[n]$ does not vary much after initial convergence while exciting the same mode. The algorithm must explicitly compute the inverse of $\mathbf{\Phi}[n]$ at each step, although $\mathbf{\Phi}[n]$ itself can still be updated recursively. Because this matrix is Hermitian and its dimension is determined by the number of elements in the source array, the inversion should be manageable for most deployments. In most environments, the time required for acoustic propagation between the source and feedback arrays, and not the computational requirements, will limit the iteration time.

The large condition numbers of $\mathbf{\Phi}[n]$ also cause the algorithm to be overly sensitive to observational noise. To alleviate this problem, the algorithm computes $(\mathbf{\Phi}[n])^{-1}$ by first performing a singular value decomposition of $\mathbf{\Phi}[n]$, then increasing any singular value less σ_1, the largest singular value, divided by Λ to be σ_1/Λ. $(\mathbf{\Phi}[n])^{-1}$ is constructed from this modified singular value decomposition, and thus has a condition number of at most Λ. However, it is important that $\mathbf{\Phi}[n]$ itself is propagated unmodified, because limiting the condition number of $\mathbf{\Phi}[n]$ will adversely effect the algorithm's transient behavior when either the channel or desired mode changes abruptly. Choosing Λ is a compromise between reducing the algorithm's sensitivity to noise and reducing its adaption time following abrupt changes.

4. ESTIMATION ERROR BOUNDS FOR TRUNCATED ARRAYS

The algorithm's ability to excite a single mode is clearly limited by the precision with which it can estimate the modes propagating at the feedback array. This requirement can conflict with practical issues of array design, deployment, and cost in many environments of interest. A truncated feedback array spanning only a portion of the water column may be greatly preferable from the point of view of these practical concerns. However, such a compromise will impact on the quality of mode estimation in most scenarios. Quantifying the degradation in performance as a function of array length with allow us to make this tradeoff in an intelligent manner.

For this section, we will assume a model where the received pressure field consists of a superposition of modes plus spatially-uncorrelated identically distributed zero-mean white Gaussian noise. Our model can be expressed by the equation

$$\mathbf{p} = \mathbf{\Psi}\mathbf{d} + \mathbf{e}, \tag{13}$$

where $\mathbf{p}$ is the vector of observed pressures at the hydrophone array, the columns of $\mathbf{\Psi}$ are the mode shapes sampled at the hydrophone locations, $\mathbf{d}$ is the vector of mode coefficients and $\mathbf{e}$ is the observation noise. For this scenario, the optimal least-squares estimate of the mode coefficients is $\widehat{\mathbf{d}} = \mathbf{\Psi}^\dagger\mathbf{p}$, where $(\cdot)^\dagger$ denotes the pseudo-inverse operator[8]. Even if $\mathbf{e}$ is not Gaussian, white, and spatially-uncorrelated, this is the best linear least-squares estimator. For the assumptions made above, this estimator is unbiased, and the covariance matrix of $\widehat{\mathbf{d}}$ is $\mathbf{\Psi}^\dagger(\mathbf{\Psi}^\dagger)^H\sigma^2$, where σ^2 is the variance of the observational noise. As the array aperture decreases, $\mathbf{\Psi}$ changes, and thus the variance of our estimates of the mode coefficients also change.

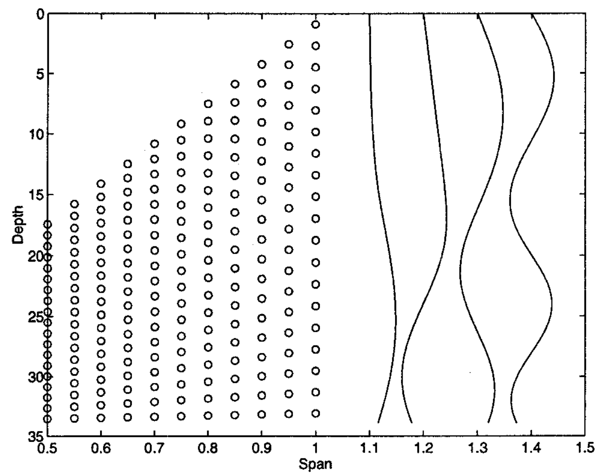

Figure 1: Hydrophone locations and mode shapes for estimator

To examine degradation in a realistic scenario, we used typical environmental data measured on the South Continental Shelf around 41°N, 71°W in August, 1993, with a frequency of 400 Hz. Under these conditions, there are nine trapped modes propagating in the channel. The variance of the mode estimators was computed for a range of array apertures. Each array contained 19 hydrophones equally spaced to cover the desired portion of the water column from the bottom upwards. Fig. 1 plots the hydrophone locations for each trial alongside the first four mode shapes. The degradation in estimate quality was computed relative to the mean performance over all modes for a fully-spanning array. Fig. 2 plots this degradation for the first four modes. It is clear that most modes pay a severe penalty in estimate fidelity for shortening the array span below 0.85. Reducing the array aperture to only half of the water column would produce estimates that were unusable in almost any realistic ocean scenario, since even the best estimator is degraded by more than 60 dB.

5. LABORATORY WAVEGUIDE EXPERIMENT

This section presents preliminary results indicating the algorithm performs well in a laboratory waveguide environment. The experiment was conducted at 8 kHz in a waveguide roughly 20 m long and 1.2 m wide, with a water depth of 0.6 m. The source array consisted of six piezoelectric underwater sources, and the feedback array had seven elements roughly equally spaced in depth. The algorithm was initialized using a random channel matrix. Figure 3 plots the ratio of the energy in the desired field to the energy in the error as a function of the iteration number. It can be seen from this figure that in spite of a very poor initial estimate of the channel, the algorithm converged within roughly fifty iterations to produce a close fit to the desired pressure field at the feedback array. Fig. 4 shows the desired and received pressure field for this experiment after the algorithm has converged. The ratio of the energy in the desired pressure field to the energy in the error is roughly 25 dB. This result suggests the algorithm should do well for any environment that can be characterized by our linear model, since the algorithm was initialized with no *a priori* knowledge of the environment.

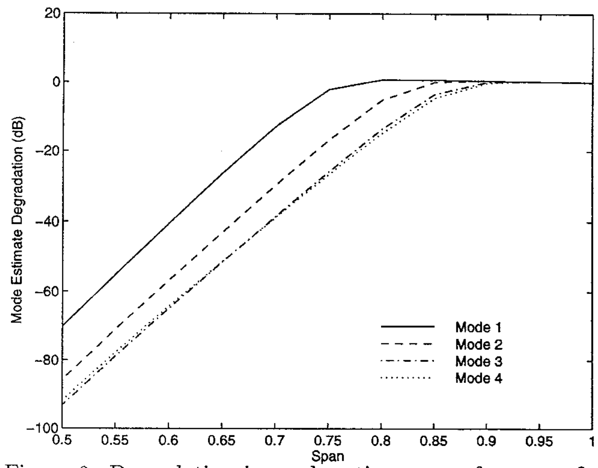

Figure 2: Degradation in mode estimator performance for decreasing array aperture.

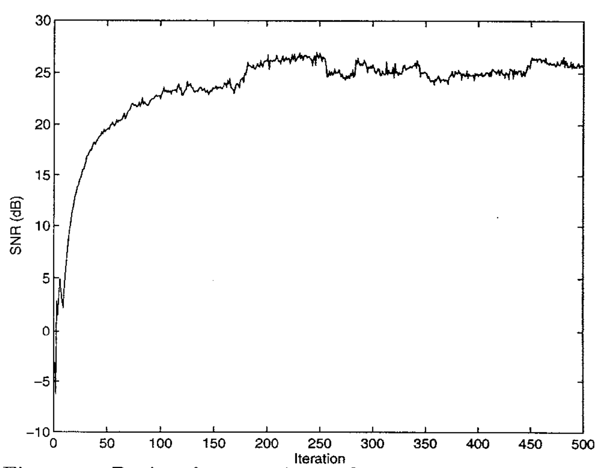

Figure 3: Ratio of energy in mode to energy in noise for laboratory waveguide.

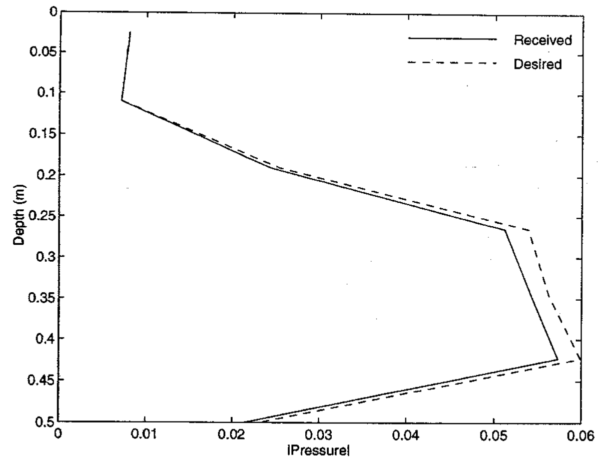

Figure 4: Pressure profile of received pressure field in laboratory waveguide after convergence.

6. REFERENCES

[1] C. S. Clay and K. Huang. Single mode transmission and acoustic backscattering measurements in a laboratory waveguide. *Journal of the Acoustical Society of America*, 67(3):792–794, March 1980.

[2] C. Gazanhes and J. L. Garnier. Experiments on single mode excitation in shallow water propagation. *Journal of the Acoustical Society of America*, 69(4):963 – 969, April 1981.

[3] C. T. Tindle, H. Hobaek, and T. G. Muir. Downslope propagation of normal modes in a shallow water wedge. *Journal of the Acoustical Society of America*, 81(2):275 – 286, February 1987.

[4] George V. Frisk. *Ocean and Seabed Acoustics: A Theory of Wave Propagation.* Prentice Hall, Englewood Cliffs, NJ, 1994.

[5] Clarence S. Clay and Herman Medwin. *Acoustical Oceanography: Principles and Applications.* Ocean Engineering. John Wiley & Sons, New York, NY, 1977.

[6] Finn B. Jensen, William A. Kuperman, Michael B. Porter, and Henrik Schmidt. *Computational Ocean Acoustics.* AIP Series in Modern Acoustics and Signal Processing. AIP Press, Woodbury, NY, 1994.

[7] Kumpati S. Narendra and Anuradha M. Annaswamy. *Stable Adaptive Systems.* Information and System Sciences Series. Prentice Hall, Englewood Cliffs, NJ, 1989.

[8] C. T. Tindle, K. M. Guthrie, G. E. J. Bold, M. D. Johns, D. Jones, K. O. Dixon, and T. G. Birdsall. Measurements of the frequency dependence of normal modes. *Journal of the Acoustical Society of America*, 64(4):1178 – 1185, October 1978.

[9] Simon Haykin. *Adaptive Filter Theory.* Information and System Sciences Series. Prentice Hall, Englewood Cliffs, NJ, second edition, 1991.