ELSEVIER

Signal Processing 82 (2002) 321–325

SIGNAL PROCESSING

www.elsevier.com/locate/sigpro

Fast communication

Orthogonal multiuser detection

Yonina C. Eldar *, Alan V. Oppenheim

Research Laboratory of Electronics, Massachusetts Institute of Technology, Room 36-615, Cambridge, MA 02139, USA

Received 31 May 2001

Abstract

In this paper we propose a new linear multiuser detector for synchronous CDMA systems. Specifically, the received signal is demodulated using an orthogonal multiuser (OMU) receiver that is matched to a set of orthogonal vectors that are closest in a least-squares sense to the signature vectors. We show that this approach is equivalent to optimally whitening the noise component in the output of the decorrelator prior to detection. We provide simulation results suggesting that in many cases the OMU detector outperforms the matched filter detector and the decorrelator. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Multiuser detection; Least squares; Whitening

1. Introduction

Multiuser detectors for the detection of CDMA signals try to mitigate the effect of multiple-access interference (MAI) and background noise. These include the optimal multiuser detector, the linear minimum mean-squared error (MMSE) detector, the decorrelator, and the matched filter (MF) detector [8].

Both the optimal detector and the linear MMSE detector require knowledge of the noise level and the received amplitudes of the users' signals. In this paper we focus our attention on linear receivers that only require knowledge of the signature vectors. Two such receivers are the MF and the decorrelator. The MF optimally compensates for the white noise, but does not exploit the structure of the MAI; the decorrelator optimally rejects the MAI but does not consider the white noise.

In this paper we propose a new linear multiuser detector, which we refer to as the orthogonal multiuser (OMU) detector. The OMU detector tries to mitigate both the effect of the MAI and the white noise, by optimally whitening [5,4] the noise component in the output of the decorrelator prior to detection. Similar ideas have been explored in the context of multi-signature detection [6,0]. We show that this approach is equivalent to correlating the received signal with a set of orthonormal vectors that are closest in a least-squares (LS) sense to the signature vectors [1]. We provide simulation results that suggest that in many cases the OMU detector outperforms the MF detector and the decorrelator.

2. Problem formulation

Consider an M-user white Gaussian synchronous CDMA system. The discrete-time model for the received signal $\mathbf{r}$ is given by

$$\mathbf{r} = \mathbf{SAb} + \mathbf{n}, \tag{1}$$

* Corresponding author.

E-mail addresses: yonina@mit.edu (Y.C. Eldar), avo@mit.edu (A.V. Oppenheim).

0165-1684/02/$ - see front matter
PII: S0165-1684(01)00192-X

where $\mathbf{S}$ is the matrix of signatures $\mathbf{s}_m \in \mathscr{C}^N$, $\mathbf{A} = \mathrm{diag}(A_1, \ldots, A_M)$ where $A_m > 0$ is the received amplitude of the mth user's signal, $\mathbf{b}$ is the data vector with elements $b_m \in \{1, -1\}$, and $\mathbf{n}$ is a noise vector whose elements are independent $\mathscr{CN}(0, \sigma^2)$. We assume that all data vectors are equally likely with covariance $\mathbf{I}$, and that $\mathbf{s}_m^* \mathbf{s}_m = 1$ for all m. For simplicity, we further assume that the vectors $\mathbf{s}_m$ are linearly independent. The more general case of linearly dependent signals is considered in [0].

Based on the observed signal $\mathbf{r}$ we design a receiver to demodulate the information transmitted by each user. We restrict our attention to linear receivers that do not require knowledge of the received amplitudes or the noise level. The simplest such receiver is the single user MF, which consists of correlating the received signal with each of the signature vectors, and then detecting the mth user's bit as $\hat{b}_m = \mathrm{sgn}(\mathbf{s}_m^* \mathbf{r})$. The MF detector optimally compensates for the white noise on the channel, however, it does not take the structure of the MAI into account.

A linear multiuser detector that exploits the MAI structure without knowledge of the channel parameters is the decorrelator [7], which consists of correlating the received signal with each of the columns $\mathbf{v}_m$ of $\mathbf{V} = \mathbf{S}(\mathbf{S}^*\mathbf{S})^{-1}$, so that $a_m = \mathbf{v}_m^* \mathbf{r}$. The mth user's bit is then detected as $\hat{b}_m = \mathrm{sgn}(a_m)$. The decorrelator optimally rejects the MAI, but does not compensate for the white noise.

It was noted in [7] that the decorrelator does not generally lead to optimal decisions, since in general the noise components in the outputs a_m of the decorrelator are correlated. This correlation is due to the fact that the outputs a_m share information regarding the noise $\mathbf{n}$. In our modification of the decorrelator we propose decorrelating the noise components in the outputs prior to detection.

Let $\mathbf{a}$ denote the vector output of the decorrelator receiver. Then,

$$\mathbf{a} = \mathbf{V}^*\mathbf{r} = \mathbf{A}\mathbf{b} + \mathbf{V}^*\mathbf{n}. \tag{2}$$

The covariance of the noise component $\mathbf{V}^*\mathbf{n}$ in $\mathbf{a}$ is equal to the covariance of $\mathbf{a} - \mathbf{a}'$ where $\mathbf{a}' = E(\mathbf{a}|\mathbf{b})$, and is given by

$$\mathbf{C}_a = \sigma^2 \mathbf{V}^*\mathbf{V} = \sigma^2 (\mathbf{S}^*\mathbf{S})^{-1}. \tag{3}$$

From (3) it follows that the noise components in $\mathbf{a}$ are uncorrelated if and only if the signature vectors $\mathbf{s}_m$ are orthogonal. In this case the decorrelator does in fact lead to optimal decisions [8]. To improve the detection performance when the signatures are not orthonormal, without estimating the variance of the noise or the received amplitudes of the user's signals, we propose whitening [1] the output of the decorrelator receiver prior to detection, as depicted in Fig. 1. We will show that this approach does lead to improved performance over the MF detector and the decorrelator in many cases.

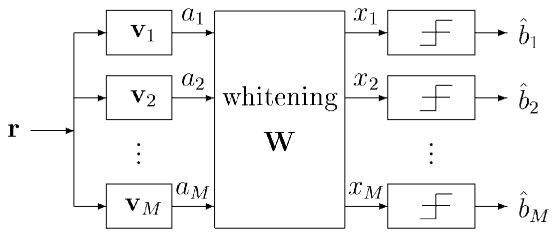

Fig. 1. Decorrelator receiver followed by whitening and detection.

Suppose we whiten the output of the decorrelator receiver $\mathbf{a}$ using a whitening transformation (WT) $\mathbf{W}$, to obtain the random vector $\mathbf{x} = \mathbf{W}\mathbf{a}$, where the covariance matrix of the noise component in $\mathbf{x}$ is given by $\mathbf{C}_x = \sigma^2 \mathbf{I}$, and then base our detection on $\mathbf{x}$. Thus the mth user's detected bit is $\hat{b}_m = \mathrm{sgn}(x_m)$. Since the detection is based on $\mathbf{x}$, we choose a WT $\mathbf{W}$ that minimizes the mean-squared error (MSE) given by

$$\varepsilon_{\mathrm{MSE}} = \sum_{m=1}^{M} E((x'_m - a'_m)^2), \tag{4}$$

where $a'_m = a_m - E(a_m|\mathbf{b})$ and $x'_m = x_m - E(x_m|\mathbf{b})$.

3. Equivalent problems

In this section we formulate the problem of (4) in two equivalent ways that provide further insight into the problem. Specifically, we show that the following problems are the same:

(1) Find an optimal WT $\mathbf{W}$ that minimizes the MSE defined by (4) between the whitened output $\mathbf{x} =$

[1] In this paper when we refer to whitening of a random vector $\mathbf{a}$ we explicitly mean whitening the noise component in $\mathbf{a}$. Equivalently, this corresponds to whitening $\mathbf{a} - E(\mathbf{a}|\mathbf{b})$. Similarly, when we say that a random vector $\mathbf{x}$ is white we explicitly mean that the noise component in $\mathbf{x}$ is white, i.e., the covariance matrix of the noise component in $\mathbf{x}$, $\mathbf{x} - E(\mathbf{x}|\mathbf{b})$, is proportional to $\mathbf{I}$.

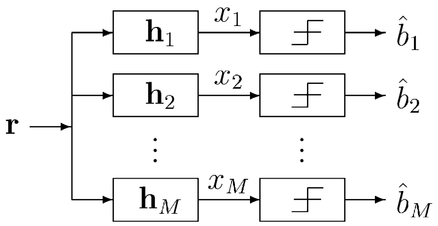

Fig. 2. Orthogonal multiuser detector.

$\mathbf{Wa}$ and the input $\mathbf{a}$, where $\mathbf{a}$ is the vector output of the decorrelator receiver.

(2) Find a set of orthonormal vectors $\{\mathbf{h}_m, 1 \leqslant m \leqslant M\}$ that are closest in an LS sense to the vectors $\{\mathbf{v}_m, 1 \leqslant m \leqslant M\}$, namely that minimize $\sum_m (\mathbf{v}_m - \mathbf{h}_m)^*(\mathbf{v}_m - \mathbf{h}_m)$. Then correlate the received signal with each of the vectors $\mathbf{h}_m$ to obtain the whitened output $\mathbf{x}$.

(3) Find a set of orthonormal vectors that are closest in an LS sense to the signature vectors $\{\mathbf{s}_m, 1 \leqslant m \leqslant M\}$. Then correlate the received signal with these vectors to obtain the whitened output $\mathbf{x}$.

In the remainder of this section we show the equivalence between the problems above, and discuss their solution. In Section 4 we briefly discuss the performance of the resulting detector, which we refer to as the orthogonal multiuser (OMU) detector.

We first show that the detector depicted in Fig. 1 is equivalent to the detector of Fig. 2, where the vectors $\mathbf{h}_m$ are *orthonormal* and are given by $\mathbf{h}_m = \sum_k \mathbf{W}^*_{km}\mathbf{v}_k$, where $\mathbf{W}^*_{km}$ denotes the kmth element of $\mathbf{W}^*$.

The output of the WT $\mathbf{x}$ in Fig. 1 is given by

$$\mathbf{x} = \mathbf{Wa} = \mathbf{WV}^*\mathbf{r} = \mathbf{H}^*\mathbf{r}, \tag{5}$$

where $\mathbf{H} = \mathbf{VW}^*$. Therefore, $\mathbf{x}$ can be viewed as the output of a bank of correlators with vectors $\mathbf{h}_m = \sum_k \mathbf{W}^*_{km}\mathbf{v}_k$, as depicted in Fig. 2. Furthermore, using (3) we have $\mathbf{H}^*\mathbf{H} = \mathbf{WV}^*\mathbf{VW}^* = 1/\sigma^2\mathbf{WC}_a\mathbf{W}^* = 1/\sigma^2\mathbf{C}_x = \mathbf{I}$, so that the vectors $\mathbf{h}_m$ are orthonormal.

We now show that minimization of $\varepsilon_{\mathrm{MSE}}$ given by (4) is equivalent to minimization of the LS error $\varepsilon_{\mathrm{LS}}(\{\mathbf{v}_m\},\{\mathbf{h}_m\})$, where

$$\varepsilon_{\mathrm{LS}}(\{\mathbf{v}_m\},\{\mathbf{h}_m\}) = \sum_{m=1}^{M}(\mathbf{v}_m - \mathbf{h}_m)^*(\mathbf{v}_m - \mathbf{h}_m). \tag{6}$$

Using (2) and (5),

$$\mathbf{x} - \mathbf{a} = (\mathbf{H} - \mathbf{V})^*\mathbf{r} = (\mathbf{H} - \mathbf{V})^*(\mathbf{SAb} + \mathbf{n}) \tag{7}$$

and $x'_m - a'_m = (\mathbf{h}_m - \mathbf{v}_m)^*\mathbf{n}$. Substituting into (4),

$$\begin{aligned}\varepsilon_{\mathrm{MSE}} &= \sum_{m=1}^{M} E((\mathbf{v}_m - \mathbf{h}_m)^*\mathbf{nn}^*(\mathbf{v}_m - \mathbf{h}_m)) \\ &= \sigma^2\sum_{m=1}^{M}(\mathbf{v}_m - \mathbf{h}_m)^*(\mathbf{v}_m - \mathbf{h}_m). \end{aligned} \tag{8}$$

Comparing (8) with (6) establishes the equivalence of problems (1) and (2).

Finally, we show that problems (2) and (3) are the same by proving that the orthonormal vectors $\mathbf{h}_m$ that minimize $\varepsilon_{\mathrm{LS}}(\{\mathbf{v}_m\},\{\mathbf{h}_m\})$ and $\varepsilon_{\mathrm{LS}}(\{\mathbf{s}_m\},\{\mathbf{h}_m\})$ are equal. To this end we rely on the following lemmas.

Lemma 1. *Let* $\{\mathbf{y}_m, 1 \leqslant m \leqslant M\}$ *be a set of orthogonal vectors with* $\mathbf{y}^*_k\mathbf{y}_m = c^2_m\delta_{km}$. *Then the orthonormal vectors* $\mathbf{h}_m$ *that minimize* $\varepsilon_{\mathrm{LS}}(\{\mathbf{y}_m\},\{\mathbf{h}_m\})$ *are* $\mathbf{h}_m = \mathbf{y}_m/|c_m|$.

Proof. Since $\mathbf{h}^*_m\mathbf{h}_m = 1$, minimization of $\varepsilon_{\mathrm{LS}}(\{\mathbf{y}_m\},\{\mathbf{h}_m\})$ is equivalent to maximization of $\sum_{m=1}^{M}\Re\{\mathbf{h}^*_m\mathbf{y}_m\}$. Using the Cauchy–Schwarz inequality,

$$\sum_{m=1}^{M}\Re\{\mathbf{h}^*_m\mathbf{y}_m\} \leqslant \sum_{m=1}^{M}|\mathbf{h}^*_m\mathbf{y}_m| \leqslant \sum_{m=1}^{M}(\mathbf{y}^*_m\mathbf{y}_m)^{1/2} \tag{9}$$

with equality if and only if $\mathbf{h}_m = \mathbf{y}_m/|c_m|$. □

As a result of Lemma 1, we have the following corollary.

Corollary 2. *Let* $\{\mathbf{y}'_m = d_m\mathbf{y}_m, 1 \leqslant m \leqslant M\}$, *where* $d_m > 0$ *are arbitrary constants and the vectors* $\mathbf{y}_m$ *are orthogonal. Then the orthonormal vectors* $\mathbf{h}_m$ *that minimize* $\varepsilon_{\mathrm{LS}}(\{\mathbf{y}_m\},\{\mathbf{h}_m\})$ *and* $\varepsilon_{\mathrm{LS}}(\{\mathbf{y}'_m\},\{\mathbf{h}_m\})$ *are the same.*

Lemma 3. *Let* $\mathbf{y}_m$ *and* $\mathbf{y}'_m$ *denote the columns of* $\mathbf{Y}$ and $\mathbf{Y}' = \mathbf{YU}$, *respectively, where* $\mathbf{U}$ *is an arbitrary unitary matrix. Let the columns of* $\mathbf{H}$ *and* $\mathbf{H}'$ *be the orthonormal vectors* $\mathbf{h}_m$ *and* $\mathbf{h}'_m$ *that minimize* $\varepsilon_{\mathrm{ls}}(\{\mathbf{y}_m\},\{\mathbf{h}_m\})$ *and* $\varepsilon_{\mathrm{ls}}(\{\mathbf{y}'_m\},\{\mathbf{h}'_m\})$, *respectively. Then* $\mathbf{H}' = \mathbf{HU}$.

Proof. Since $(\mathbf{H}')^*\mathbf{H}' = \mathbf{U}^*\mathbf{H}^*\mathbf{HU} = \mathbf{I}$, the vectors $\mathbf{h}'_m$ are orthonormal. The lemma then follows from

$$\varepsilon_{ls}(\{\mathbf{y}_m\},\{\mathbf{h}_m\}) = \mathrm{Tr}((\mathbf{Y}-\mathbf{H})^*(\mathbf{Y}-\mathbf{H}))$$
$$= \mathrm{Tr}(\mathbf{U}(\mathbf{Y}-\mathbf{H})^*(\mathbf{Y}-\mathbf{H})\mathbf{U}^*)$$
$$= \varepsilon_{LS}(\{\mathbf{y}'_m\},\{\mathbf{h}'_m\}). \quad \Box \tag{10}$$

Combining Corollary 2 and Lemma 3 it follows that if we find a unitary matrix such that the columns of $\mathbf{V}' = \mathbf{VU}$ and $\mathbf{S}' = \mathbf{SU}$ are both orthogonal and proportional to each other, then the orthonormal vectors minimizing $\varepsilon_{LS}(\{\mathbf{v}_m\},\{\mathbf{h}_m\})$ and $\varepsilon_{LS}(\{\mathbf{s}_m\},\{\mathbf{h}_m\})$ are the same. Let $\mathbf{S} = \mathbf{Q}\boldsymbol{\Sigma}\mathbf{Z}^*$ be the SVD of $\mathbf{S}$, where $\mathbf{Q}$ and $\mathbf{Z}$ are unitary matrices and $\boldsymbol{\Sigma}$ is a diagonal $N \times M$ matrix with diagonal elements $\sigma_m > 0$. Then $\mathbf{V} = \mathbf{S}(\mathbf{S}^*\mathbf{S})^{-1} = \mathbf{Q}\tilde{\boldsymbol{\Sigma}}\mathbf{Z}^*$, where $\tilde{\boldsymbol{\Sigma}}$ is a diagonal $N \times M$ matrix with diagonal elements $1/\sigma_m$. Now, let $\mathbf{V}' = \mathbf{VZ}$ and $\mathbf{S}' = \mathbf{SZ}$. Then the columns $\mathbf{v}'_m$ and $\mathbf{s}'_m$ of $\mathbf{V}'$ and $\mathbf{S}'$, respectively, are both orthogonal, and $\mathbf{v}'_m = d_m\mathbf{s}'_m$ where $d_m = 1/\sigma_m^2$. Thus, the orthonormal vectors minimizing $\varepsilon_{LS}(\{\mathbf{v}_m\},\{\mathbf{h}_m\})$ and $\varepsilon_{LS}(\{\mathbf{s}_m\},\{\mathbf{h}_m\})$ are the same.

This completes the proof that the three problems outlined at the beginning of this section are equivalent. The optimal whitening problem has been solved in its most general form in [5,0], from which it follows that the WT minimizing (4) is

$$\mathbf{W} = \sigma\mathbf{C}_a^{-1/2} = (\mathbf{S}^*\mathbf{S})^{1/2}. \tag{11}$$

The orthonormal vectors that minimize $\varepsilon_{LS}(\{\mathbf{v}_m\},\{\mathbf{h}_m\})$ and $\varepsilon_{LS}(\{\mathbf{s}_m\},\{\mathbf{h}_m\})$ are then the columns of

$$\mathbf{H} = \mathbf{VW}^* = \mathbf{V}(\mathbf{S}^*\mathbf{S})^{1/2} = \mathbf{S}(\mathbf{S}^*\mathbf{S})^{-1/2}. \tag{12}$$

We note that this solution has been obtained in the context of quantum detection [3] and in the context of general inner product shaping [1].

4. Illustration of performance

In Fig. 3, the bit-error rate in the infinite-user limit for the OMU detector is compared to the single-user MF, the decorrelator, and the linear MMSE detector, for $\beta = M/N = 0.95$. The elements of the signature matrix $\mathbf{S}$ are mutually independent $\mathcal{CN}(0, 1/N)$. For the SNR range shown, the OMU detector performs better than the decorrelator and the MF. At low SNR, the performance of the OMU detector is close to that of the linear MMSE receiver.

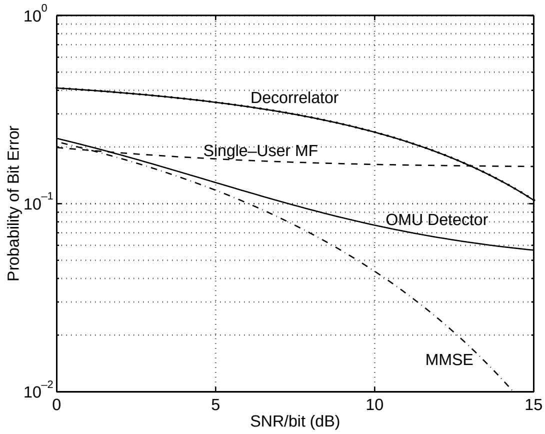

Fig. 3. Probability of bit error in the large-system limit, with equal-power users, random signatures, and $\beta = 0.95$.

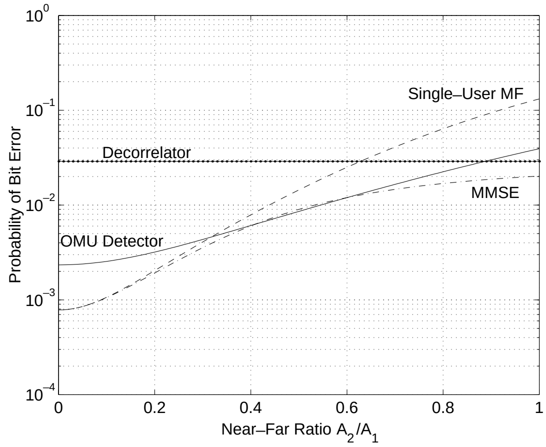

Fig. 4. Probability of bit error for two users and cross-correlation $\rho = 0.8$. The SNR of the desired user is 10 dB.

Fig. 4 evaluates the theoretical probability of bit error of the OMU detector in the special case of two users with cross-correlation $\rho = 0.8$, where the desired user has an SNR of 10 dB. The probability of bit error of the OMU detector is plotted as a function of the near–far ratio A_2/A_1, where A_1 is the amplitude of the desired user. The corresponding curves for the single-user MF, decorrelator, and linear MMSE detectors are plotted for comparison. We see that the OMU detector performs better than the decorrelator when the interferer power is roughly less than the power of the desired user. When the power of the interferer is negligible, the MF performs better than the OMU detector which is expected since the MF is optimal in the absence of MAI. Thus, the OMU detector performs better than both the decorrelator and the MF when A_2/A_1 is roughly between 0.3 and 0.9. In this regime, the OMU detector performs similarly to the linear MMSE receiver.

More extensive simulation and analysis are presented in [0].

References

[0] Y.C. Eldar, Quantum Signal Processing Ph.D. Thesis, Massachusetts Institute of Technology, December 2001.
[1] Y.C. Eldar, Least-squares inner product shaping, Linear Algebra Appl., to appear.
[3] Y.C. Eldar, G.D. Forney Jr., On quantum detection and the square-root measurement, IEEE Trans. Inform. Theory 47 (March 2001) 858–872.
[4] Y.C. Eldar, A.V. Oppenheim, Orthogonal matched filter detection, Proceedings on the International Conference Acoustics Speech, Signal Processing (ICASSP-2001), Salt Lake, UT, May 2001.
[5] Y.C. Eldar, A.V. Oppenheim, MMSE whitening and subspace whitening, IEEE Trans. Signal Processing, July 2001, submitted for publication.
[6] Y.C. Eldar, A.V. Oppenheim, D. Egnor, Orthogonal and projected orthogonal matched filter detection, IEEE Trans. Signal Processing, January 2001, submitted for publication.
[7] R. Lupas, S. Verdu, Linear multiuser detectors for synchronous code-division multiple-access channels, IEEE Trans. Inform. Theory 35 (January 1989) 123–136.
[8] S. Verdu, Multiuser Detection, Cambridge University Press, New York, 1998.