

Channel Equalization For Communication with Chaotic Signals[‡]

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Abstract. Chaotic signals and synchronized systems are potentially useful as modulation or masking waveforms for communications systems. This paper explores the effect on synchronization of frequency dependent channel gain, *i.e.* linear filtering. Several approaches to compensating for these effects are also discussed.

I. INTRODUCTION

Over the past several years there has been considerable interest in utilizing chaotic signals for spread-spectrum communications. Most strategies that have been proposed exploit the self-synchronization property of a class of chaotic systems. By necessity, synchronization of the receiver requires that the received drive signal be undistorted or that it first be appropriately equalized in amplitude, spectral content and phase. Specifically, it can be anticipated that a realistic transmission channel will introduce a time varying attenuation due to fading, scattering, etc., will modify the spectral characteristics of the transmitted signal due to channel filtering and multipath and will introduce additive noise. The effects of additive noise on synchronization have been discussed in [1]. In this paper we propose a number of techniques for estimating and compensating for the effects of the channel on amplitude, spectral content

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and phase of the synchronizing drive signal. In Section II we briefly review the various approaches that have been proposed for exploiting synchronized chaotic systems for communications. In Section III we incorporate into the system description the presence of a channel. Section IV describes the effect of channel gain on synchronization error and our approach and results for gain compensation. Section V illustrates the effect of channel filtering and proposes an approach to estimating the frequency response of an equalizing filter.

II. COMMUNICATIONS USING SYNCHRONIZED CHAOTIC SYSTEMS

Since the discovery that two chaotic systems can be made to synchronize when the second system (receiver) is driven by the first (transmitter) [2,3], researchers have been actively exploring how this characteristic might be used for practical applications. Many techniques have now been proposed for combining information-bearing signals with a chaotic carrier and for recovering the information with a self-synchronizing receiver system. One of the first demonstrations of how these concepts might be used for private communications is given in [4]. In that paper, the notion of cascaded chaotic subsystems is used to numerically demonstrate that low-level information-bearing signals can be added to a more powerful chaotic drive signal and recovered at the receiver by exploiting the robust synchronization and signal recovery properties of the Lorenz system. Later, it was shown that this chaotic signal masking and recovery concept could be implemented with standard analog hardware [5–7] and could be made to work with other self-synchronizing chaotic systems [8].

Recognizing some potential drawbacks of this approach in realistic communication scenarios, researchers developed alternative ways for privately communicating information with chaotic signals and systems. For example, by modulating a transmitter parameter with a binary-valued bit stream, it was demonstrated that digital signals can be privately transmitted and recovered with a self-synchronizing receiver system [5,6,9]. This concept has been recently extended to allow multiple independent messages to be privately communicated [10]. Other somewhat more sophisticated modulation techniques are based on chaotic “spread-spectrum” concepts, *i.e.* the information signal is directly multiplied by the broadband chaotic drive signal [11,12]. Note that each of these communication strategies have one important feature in common—they utilize the self-synchronization property of properly matched chaotic receivers to readily recover the information.

Chaotic communication methods have also been proposed that relax the requirement for synchronization at the intended receivers to readily recover the information-bearing signals. One approach utilizes the information signal to modulate the parameter(s) of a chaotic map; the information is then recovered

at the intended receivers by inverting the map with respect to the modulation parameter(s) [13]. Another approach is based on “chaotic switching,” *i.e.* information is transmitted by switching between different chaotic systems in the transmitter and detected at the receiver via maximum likelihood state estimation techniques [14]. Information can also be privately communicated with chaotic maps by altering the known unstable periodic orbits in a way that depends on whether a “zero” or “one” is to be transmitted [15]. In order for any of these chaotic communication methods to work well in practice, however, undesirable channel effects must be equalized in order to accurately recover the information signals at the intended receivers. Below, we address this issue by developing some straightforward channel equalization methods for chaotic signals.

III. SYSTEM DESCRIPTION INCORPORATING CHANNEL EFFECTS

The general structure for a synchronizing chaotic transmitter and receiver with channel effects and channel compensation is shown in Fig. 1. In the system of Fig. 1, $x(t)$ is referred to as the chaotic drive signal and $x_r(t)$ as the reconstructed drive signal. In the absence of transmission channel effects, $\tilde{x}(t)$ and $x(t)$ are identical and no compensator is required. In that case $s(t) = x(t)$ and for systems that have the self-synchronizing property the reconstructed drive signal $x_r(t)$ will equal $s(t)$ after an initial transient. In the presence of channel effects and inexact channel compensation the receiver won't synchronize and $x_r(t)$ will not equal $s(t)$, resulting in synchronization error, defined as

$$e_x(t) \triangleq s(t) - x_r(t) . \quad (1)$$

In this paper we propose several approaches to utilizing $s(t)$ or the synchronization error to adaptively determine an appropriate compensator. As a measure of synchronization error we will make reference to P_x , $P_{\tilde{x}}$, and P_{e_x} representing the time-average power over some appropriately chosen time window in the chaotic drive signal $x(t)$ the received signal $\tilde{x}(t)$ and the synchronization error, respectively. We assume that for any chaotic system exhibiting steady state behavior on the attractor, P_x is a quantity that can be determined apriori either empirically or analytically.

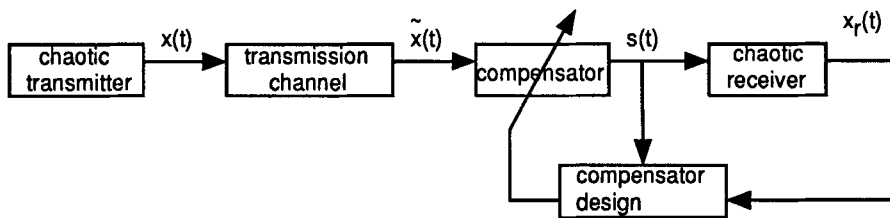


FIG. 1. General structure for a synchronizing chaotic transmitter and receiver with channel effects and channel compensation.

In the system of Fig. 1 and throughout our discussion in this paper we are not incorporating any specific modulation strategy utilizing the chaotic transmitter and receiver. Our specific focus here is on determining a compensator assuming only that the chaotic drive signal has been modified during transmission through a channel that can be represented by a linear system whose characteristics may be slowly time varying.

In this paper we propose and describe a number of approaches to determining the channel compensation. While the approaches are not specific to any particular chaotic system, we have used the Lorenz transmitter-receiver system as the prototype for this study. Within the Lorenz framework, the transmitter and receiver equations used are

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= rx - y - xz \\ \dot{z} &= xy - bz \quad , \end{aligned} \quad (2)$$

$$\begin{aligned} \dot{x}_r &= \sigma(y_r - x_r) \\ \dot{y}_r &= rs(t) - y_r - s(t)z_r \\ \dot{z}_r &= s(t)y_r - bz_r \quad , \end{aligned} \quad (3)$$

where $s(t)$ = drive signal in the receiver equations,
 x, y, z = transmitter state variables,
 x_r, y_r, z_r = receiver state variables.

The analysis of these equations with regard to synchronization of the transmitter and receiver is given in [5-7].

The specific parameter values used were $\sigma = 16, r = 45.6,$ and $b = 4$. For the simulations presented in this paper, the Lorenz equations were numerically integrated using a fourth-order Runge-Kutta algorithm with a time step of .005.

IV. CHANNEL GAIN COMPENSATION

In this section we assume that the channel imposes only a constant or time-varying gain on the transmitted drive signal $x(t)$, *i.e.* the received signal is $\tilde{x}(t) = G(t)x(t)$. If $G(t)$ differs from unity, then synchronization deteriorates, as illustrated Fig. 2. Specifically, Fig. 2 shows the synchronization error power P_{ex} with no channel compensation, *i.e.* $s(t) = \tilde{x}(t)$, and for constant channel gain or attenuation for a range of channel gain variation. As we expect, P_{ex} is essentially zero for $G = 1$ and rapidly increases as the channel introduces non-unity gain or attenuation.

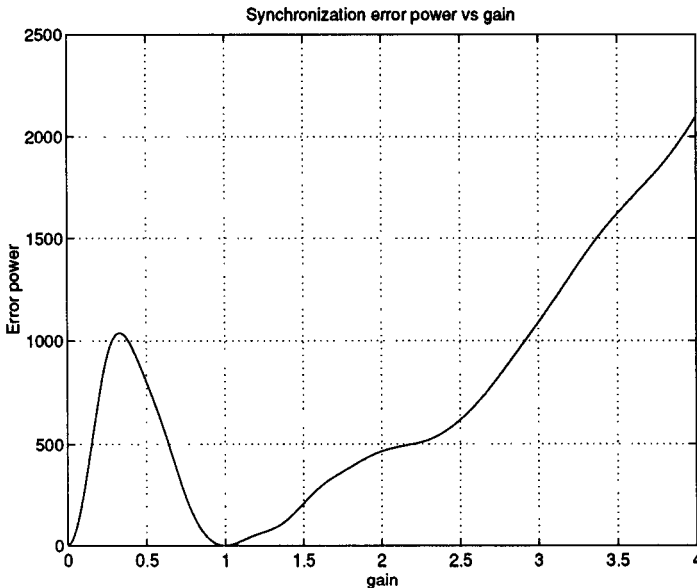


FIG. 2. Synchronization error power vs. gain.

There are several potential approaches to estimating the channel gain or attenuation so that a compensating gain can be applied. From Fig. 2 we see that the synchronization error power is unimodal for a range of gains around unity. Consequently, one approach is to iteratively adjust the compensating gain to minimize P_{ex} . This assumes that an initial estimate of the gain is within the range in Fig. 2 which is unimodal and includes $G = 1$ and for which the slope of the curve is sufficiently high to permit a reasonable gradient search for the gain.

A reasonable initial estimate of the gain can be obtained by utilizing the fact that P_x , the power in the chaotic drive signal at the input to the channel, can be expected to be independent of the specific sample path. Empirical measurement of P_x over a range of initial conditions in the Lorenz system with

500 independent trials and a time window of 800 sec resulted in an average value of 159.78, a variance of .018, a maximum of 160.16 and a minimum of 159.45. Based on this, a reasonable initial estimate of $\hat{G}(t)$ is then taken as

$$\hat{G} = \sqrt{\frac{P_{\hat{x}}(t)}{P_x}} = \sqrt{\frac{P_{\hat{x}}(t)}{159.78}} \quad (4)$$

A rectangular sliding window of duration Δ is used to estimate $P_{\hat{x}}$, *i.e.*

$$\hat{P}_{\hat{x}}(t) = \frac{1}{\Delta} \int_{t-\frac{\Delta}{2}}^{t+\frac{\Delta}{2}} \hat{x}^2(\tau) d\tau \quad (5)$$

In Fig. 3 we show a representative example of compensation for a constant gain $G(t) = 3$. In this figure we show the synchronization error when the compensating gain is the reciprocal of the initial gain estimate $\hat{G}(t)$ in Eq. (4). We also show on the same graph the synchronization error when for each time window, a gradient search is applied to minimize P_{ex} , starting with the initial estimate in Eq. (4). A 20-second sliding window was used to estimate $P_{\hat{x}}$. For time scale comparison, a representative 20-second segment of $x(t)$ is shown in Fig. 4. In Fig. 3(a) the initial error represents the error transient prior to synchronization. In Fig. 3(b) we show the error on an expanded scale for a 10-second interval after the initial transient. Clearly the gradient search following the initial gain estimate results in significant reduction in synchronization error.

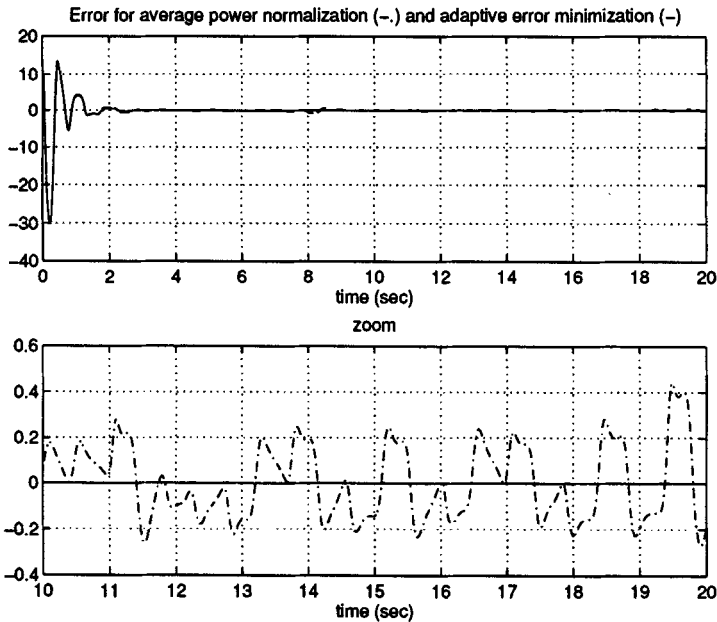


FIG. 3. Receiver's synchronization error for average power normalization and adaptive error normalization.

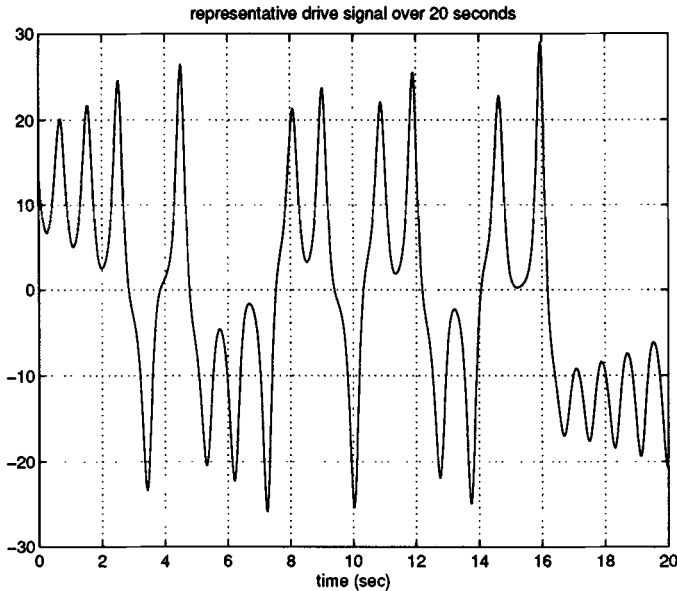


FIG. 4. Representative drive signal over a 20-second time interval.

In the example represented by Fig. 3, the channel gain was constant. In Fig. 5 we show corresponding results with a linearly time varying gain

$$G(t) = 0.95 + (t/2000) . \quad (6)$$

A 20-second window is again used to estimate $P_{\bar{x}}(t)$. Superimposed in Fig. 5 are the instantaneous error using only the initial gain estimate and using a gain estimate based on applying a gradient search to minimize P_{ex} within each window. In Fig. 5, the smaller instantaneous error curve represents the compensation resulting from adaptive error minimization and the larger error curve the compensation utilizing only the initial estimate of Eq. (4). While for this particular example there is some modest improvement with the adaptive error minimization it is not as significant as the case of constant channel gain. The difficulty in using adaptive error minimization when the gain is time varying lies in the fact that the procedure relies on Fig. 1 which assumes constant gain and an accurate estimate of synchronization error power. With the gain time varying, P_{ex} must be estimated over a finite duration window and consequently will inherently have some variance. Furthermore the gain varies over the window duration. Empirically we've found that the adaptive error minimization algorithm is sensitive to time varying channel gain and reduces the synchronization error over the use of the initial gain estimate in Eq. (4) only if the variance in gain over the window is small relative to the variance in the estimate of $P_{ex}(t)$. We are currently also exploring a number of other ways of estimating and compensating for time varying channel gain.

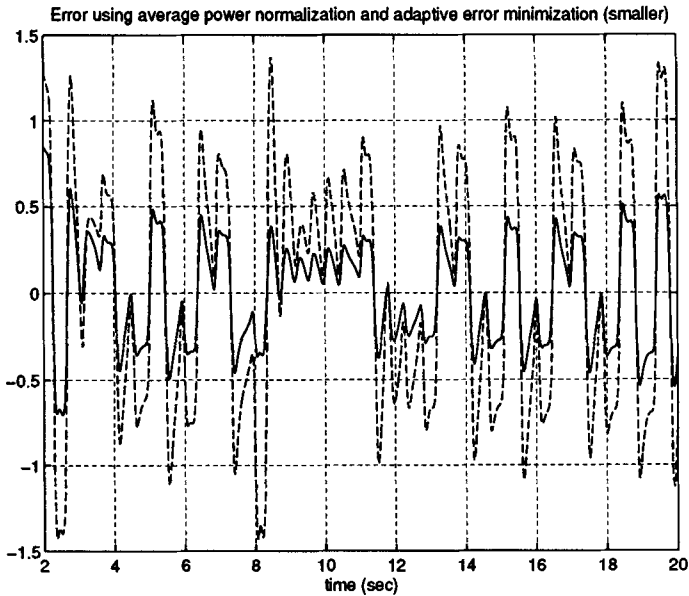


FIG. 5. Receiver's synchronization error using average power normalization (dashed) and adaptive error minimization (solid) for a time varying gain channel.

V. THE EFFECTS OF FILTERING

A realistic channel can also be expected to modify the spectral characteristics of the drive signal due to such effects as multipath, frequency-dependent attenuation etc. As with gain compensation, our proposed approach to compensating for channel filtering is to assume that the power density spectrum and higher-order power spectra of the chaotic drive signal are essentially independent of the specific sample path. The channel equalizer is then designed so that the spectral characteristics of $s(t)$ match those expected for the chaotic drive. For example, let $S_{\tilde{x}}(f)$ denote the power density spectrum of the received drive signal and $S_x(f)$ denote the power density spectrum of a representative drive signal for the transmitter. As with the calculation in Eq. (4), we assume that the power density spectrum for the specific drive signal that generated the received $\tilde{x}(t)$ will be $S_x(f)$, *i.e.* that $S_x(f)$ is independent of the specific sample path $x(t)$. With $H(f)$ denoting the frequency response of the channel,

$$S_{\tilde{x}}(f) = |H(f)|^2 S_x(f) . \quad (7)$$

Since $S_x(f)$ is known and $S_{\tilde{x}}(f)$ is estimated from the received data, $|H(f)|^2$ can be estimated. If the channel is assumed to be minimum phase, then $H(f)$ can be constructed from $|H(f)|^2$ using any of a variety of techniques such as spectral factorization, the complex cepstrum or the Hilbert transform.

To illustrate the loss of synchronization due to filtering and estimation of $|H(f)|^2$, consider the minimum phase first order filter with transfer function

$$H(s) = \frac{2.3}{s + 1} . \quad (8)$$

In Fig. 6 we show the frequency response magnitude and group delay for the filter. As indicated in Fig. 7, the significant spectral content of the Lorenz drive signal $x(t)$ extends well beyond 2 Hz, *i.e.* the one pole filter alters the spectral content significantly. In Fig. 8 we illustrate the effect of filtering on the synchronization. Specifically, Fig. 8(b) shows the instantaneous synchronization error when the channel is represented by the single pole filter and no compensation is applied. For comparison, in Fig. 8(a) we show the synchronization error with no channel distortion.

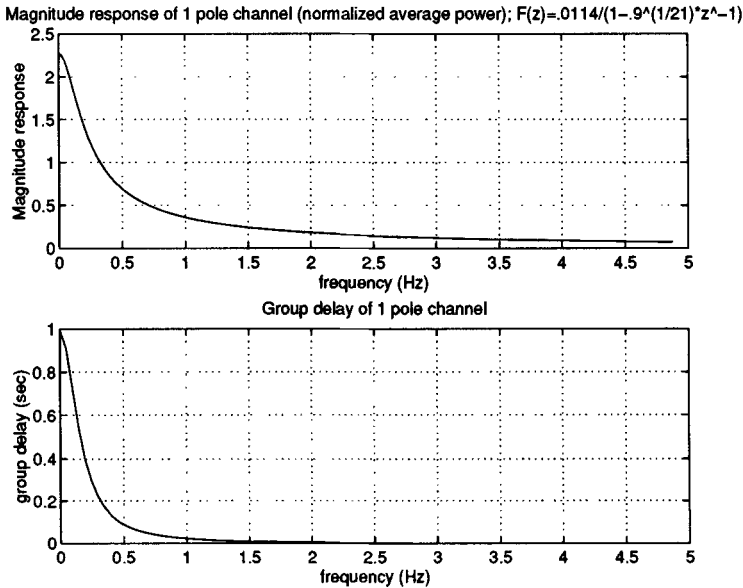


FIG. 6. Frequency response of a single pole channel filter. (a) magnitude response. (b) group delay.

For this example, $|H(f)|$ was estimated through the use of Eq. (7). The result is shown in Fig. 9 overlaid with the exact frequency response magnitude corresponding to Eq. (8). We see that for this simple and noise-free case, we are able to recover $|H(f)|$ over a much wider frequency range than would be required. We also note from Fig. 7 that since $S_x(f)$ is down by 100 dB at 10 Hz from its peak, any realistic noise in the channel would preclude estimating $H(f)$ out to even this frequency.

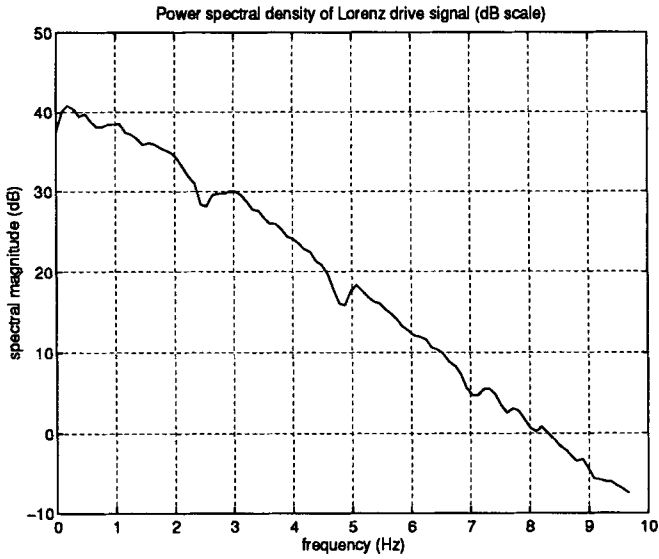


FIG. 7. Power spectral density of the Lorenz drive signal (dB scale).

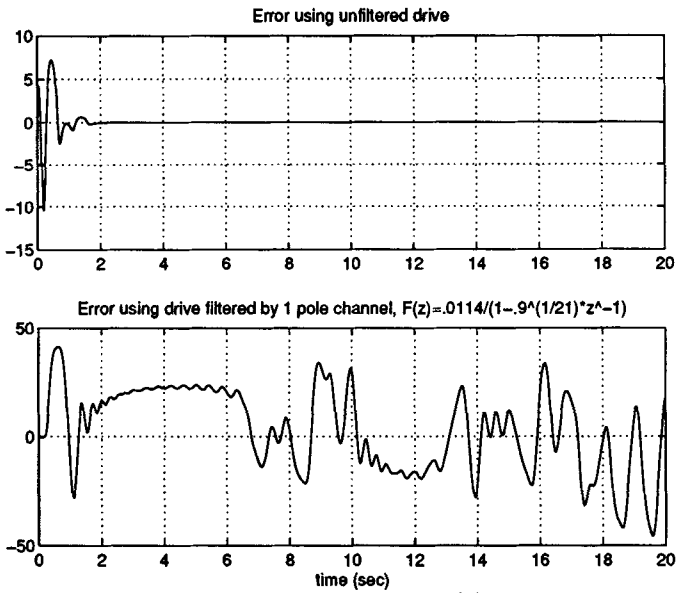


FIG. 8. Receiver's synchronization error using: (a) unfiltered drive signal; and (b) drive signal filtered with a single pole channel filter.

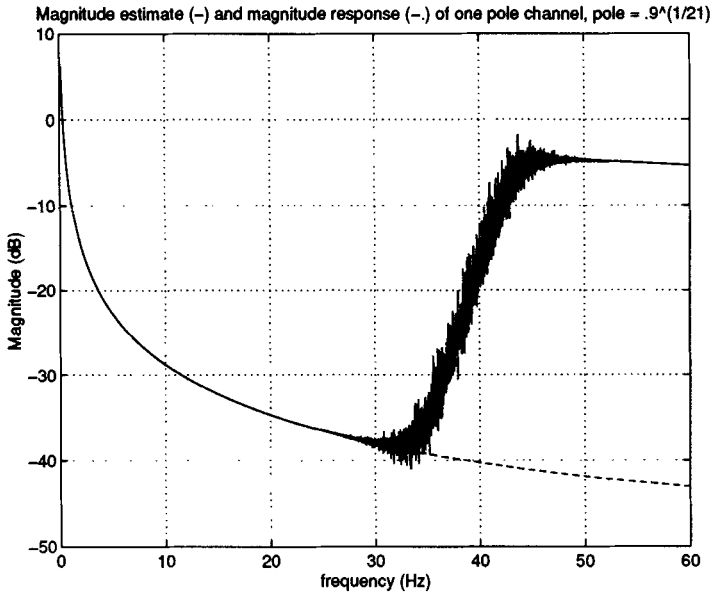


FIG. 9. Magnitude estimate (solid) and magnitude response (dashed) of a single pole channel filter.

As a second example of the effect on synchronization of channel filtering, we consider an all-pass channel with transfer function

$$H(s) = \frac{s + .995}{s - .995} . \quad (9)$$

For this example, $|H(f)|^2 = 1$. The group delay for this channel is shown in Fig. 10. The instantaneous synchronization error is shown in Fig. 11(b). Again, for comparison, the synchronization error with the correct drive signal is shown in Fig. 11(a). As this example clearly demonstrates, exact compensation of the magnitude of the frequency response is not sufficient to eliminate synchronization error since the error in Fig. 11(b) is due entirely to distortion introduced in the drive signal by the group delay. We are currently developing and exploring several approaches to estimating and compensating for the group delay characteristics of the channel as well as the frequency response magnitude.

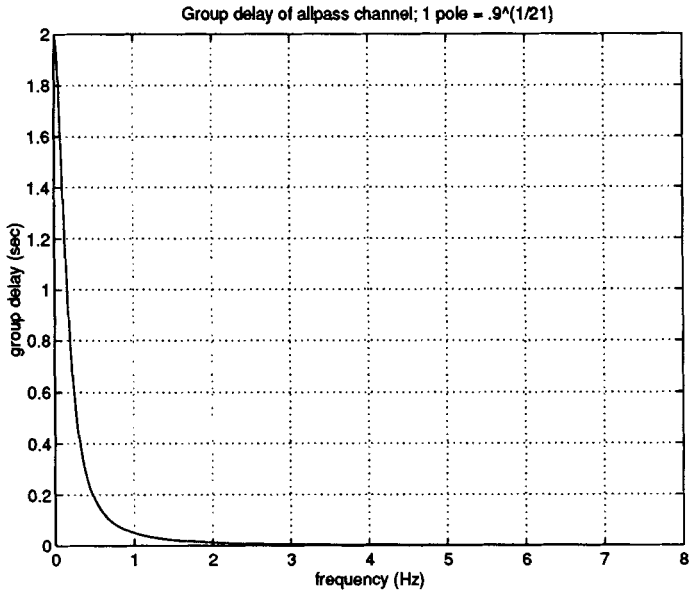


FIG. 10. Group delay of a single pole-zero allpass channel.

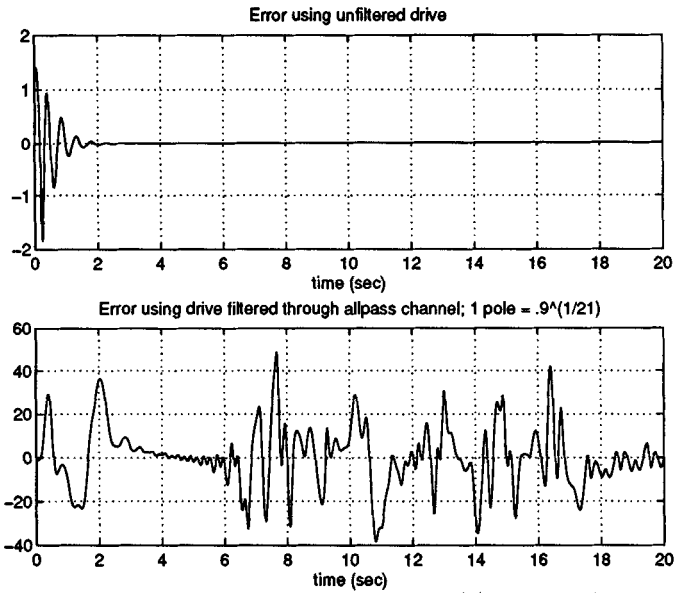


FIG. 11. Receiver's synchronization error using: (a) unfiltered drive signal; and (b) drive signal filtered with a single pole-zero allpass channel filter.

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