Homomorphic Analysis of Speech

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Abstract—**Classes of systems which satisfy a generalized principle of superposition have been previously proposed and termed "homomorphic systems," emphasizing their interpretation as homomorphic (i.e., algebraically linear) transformations. One such class appears suited to the separation of signals that have been convolved. In this paper, an approach to deconvolution of speech, based on these ideas, is discussed.**

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Introduction

IT IS GENERALLY accepted that a reasonable model for the speech waveform consists of a convolution of components representing the contributions of vocal cord timing, glottal pulse, and vocal tract impulse response. Both for speech bandwidth compression and for basic studies of the nature of the speech wave, it is desirable to isolate the effects of each of these components.

In this paper, a procedure for separating the components of speech is proposed and discussed. The procedure is based on an approach to nonlinear filtering of signals which have been nonadditively combined, that has been termed generalized linear filtering. In its realization for the deconvolution of speech, it is similar in some respects to cepstral analysis,[1] with the primary difference resulting from its application to the separation of the components rather than detection. The motivation for applying these ideas to speech processing was a direct result of the success of cepstral pitch detection, as discussed by Noll.[2]

Homomorphic Deconvolution

Consider a signal $s(t)$ consisting of the convolution of two components $s_1(t)$ and $s_2(t)$ so that $s(t)=s_1(t)\otimes s_2(t)$, where $\otimes$ denotes convolution. In a manner similar to the linear filtering problem, we can restrict the class of filters to those having the property that, if ϕ denotes the transformation of the filter, then

$$\phi[s_1(t) \otimes s_2(t)] = \phi[s_1(t)] \otimes \phi[s_2(t)]. \qquad (1)$$

In other words, the filter satisfies a principle of superposition under an operation (convolution) which is matched to the way in which the signals to be separated have been combined, in the same way that linear filters are matched to signals which have been added. The general class of systems satisfying a principle of superposition under some rule of combination for the inputs and outputs, has been termed homomorphic systems.[3] The particular class of homomorphic systems represented by (1) has been studied in detail. A canonic representation for this class of filters is shown in Fig. 1, in which

Fig. 1. Canonic form for homomorphic deconvolution.

the system D is invertible and has the property that

$$D[s_1(t) \otimes s_2(t)] = D[s_1(t)] + D[s_2(t)] \qquad (2)$$

and the system D^{-1} is the inverse of the system D. The system L is a linear system. Thus, any system having the property specified by (1) can be decomposed in the form of Fig. 1, and any system of this form will have the property of (1).

There are several possible equivalent representations of the system D. The most straightforward and most generally applicable is shown in Fig. 2. In this representation, the response $\hat{s}(t)$ of the system D is related to the excitation $s(t)$ through the equation

$$S(j\omega) = \log S(j\omega) = \log |S(j\omega)| + j\theta(j\omega)$$

where $S(j\omega)$ and $\hat{S}(j\omega)$ are the complex Fourier transforms of $s(t)$ and $\hat{s}(t)$, respectively, and $\theta(j\omega)$ is the phase associated with $S(j\omega)$. We note that $\hat{s}(t)$ is similar to the cepstrum in that it results from a spectral transformation on the log spectrum. The differences result from the fact that the cepstrum incorporates only spectral magnitude information, whereas $\hat{s}(t)$ uses both spectral magnitude and phase. For this reason, it has been convenient to refer to $\hat{s}(t)$ as the complex cepstrum, emphasizing the use of the complex Fourier transform and the complex logarithm.

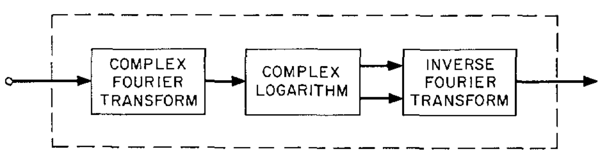

Fig. 2. Realization of the transformation $D(\cdot)$ of Fig. 1.

Properties of the Complex Cepstrum

In realizing the transformation D, it is necessary to use a digital computer; consequently, the input and output are viewed as discrete sequences, and the Fourier transformation is replaced by the z-transform evaluated on the unit circle. Thus, we will represent the input sequence as $s(n)$, the complex cepstrum as $\hat{s}(n)$, and their z-transforms as $S(z)$ and $\hat{S}(z)$, respectively, so that

$$\hat{S}(z) = \log S(z).$$

The phase associated with $S(z)$ evaluated on the unit circle is considered as a continuous and odd function of ω in the range $-\pi < \omega < \pi$.

If we restrict $S(z)$ to be of the form

$$S(z) = |k| z^r \frac{\prod_{i=1}^{m_0} (1 - a_i z^{-1}) \prod_{i=1}^{m_1} (1 - b_i z)}{\prod_{i=1}^{p_0} (1 - c_i z^{-1}) \prod_{i=1}^{p_1} (1 - d_i z)}$$

where a_i and c_i are the zeros and poles, respectively, inside the unit circle, and $(1/b_i)$ and $(1/d_i)$ are the zeros and poles respectively outside the unit circle, then it can be shown[1] that

$$\hat{s}(n) = \frac{r}{n} \cos \pi n - \frac{1}{2\pi j n} \oint_c \left[z \frac{S'(z)}{S(z)} \right] z^{n-1} dz \quad n \neq 0 \qquad (3)$$

[1] The complex cepstrum $\hat{s}(n)$ is given by the inverse z-transform of $\log S(z)$, i.e., $\hat{s}(n) = (1/2\pi j) \oint \log S(z) z^{n-1}\, dz$. Equation (3) is derived by integrating this equation by parts for z on the unit circle. Since the phase is considered as a continuous and odd function of ω, it is discontinuous at $\omega = n\pi$ if $r \neq 0$, resulting in the first term in (3).

where the contour of integration c is taken to be the unit circle. If $s(n)$ has no poles or zeros outside the unit circle (including poles or zeros at infinity), corresponding to an input sequence which is minimum phase, then $r = 0$ and (3) becomes

$$\hat{s}(n) = \frac{1}{2\pi j n} \oint \left[-z \frac{S'(z)}{S'(z)} \right] z^{n-1} dz \qquad n \neq 0. \qquad (4)$$

However, the poles of the factor $z(S'(z)/S(z))$ occur at values of z for which $S(z)$ has either poles or zeros, which, for the case of a minimum phase sequence, are entirely within the unit circle in the z-plane. Under this condition, then, $\hat{s}(n) = 0$ for $n < 0$; in other words, the complex cepstrum of a minimum phase sequence is zero for $n < 0$. In a similar manner, we can consider sequences for which $S(z)$ has all its poles and zeros outside the unit circle. Such sequences could appropriately be termed maximum phase sequences. In this case, the complex cepstrum is zero for $n > 0$.

Because of the restriction placed on the phase of $S(z)$, namely, that it is an odd function of ω, the complex cepstrum will always have zero imaginary part, i.e., it is always a pure real function. This is seen by observing that

$$\hat{s}(n) = \frac{1}{2\pi j} \oint \log S(z) z^{n-1} dz$$

or, with the contour of integration taken to be the unit circle,

$$\hat{s}(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} [\log |S(e^{j\omega})| + j\theta(\omega)] e^{j\omega n} d\omega.$$

With $\log |S(e^{j\omega})|$ an even function of ω, and $\theta(\omega)$ an odd function of ω,

$$\hat{s}(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log |S(e^{j\pi})| \cos \omega n d\omega - \frac{1}{2\pi} \int_{-\pi}^{\pi} \theta(\omega) \sin \omega n d\omega.$$

If the sequence $s(n)$ is minimum phase, then it is possible to obtain the complex cepstrum from the inverse transform of the logarithm of the spectral magnitude. Let $EV[\hat{s}(n)]$ denote the even part of $\hat{s}(n)$, i.e.,

$$EV[\hat{s}(n)] = \tfrac{1}{2}[\hat{s}(n) + \hat{s}(-n)].$$

Since $\hat{s}(n)$ is zero for $n < 0$ if $s(n)$ is minimum phase, then

$$\hat{s}(n) = 2u(n) EV[\hat{s}(n)]$$

where

$$u(n) = \begin{cases} 1 & n > 0 \\ \frac{1}{2} & n = 0 \\ 0 & n < 0 \end{cases}$$

but

$$EV[\hat{s}(n)] = \mathcal{Z}^{-1}[\log|S(e^{j\omega})|] \quad (5)$$

and, therefore,

$$\hat{s}(n) = 2u(n)\cdot\mathcal{Z}^{-1}[\log|S(e^{j\omega})|] \quad (6)$$

where $\mathcal{Z}^{-1}$ denotes the inverse z-transform.

Determining the complex cepstrum on the basis of (6) corresponds to reconstructing the phase associated with the magnitude of the spectrum by using the Hilbert transform. We observe also from (5) that $EV[\hat{s}(n)]$ corresponds to the cepstrum.[2] For minimum phase sequences, the cepstrum and the complex cepstrum will be identical (except for a factor of 2) for $n>0$.

The Complex Cepstrum of Speech

We assume that samples of the speech waveform can be considered as the discrete convolution of sequences representing pitch, glottal pulse, and vocal tract, denoted by $p(n)$, $g(n)$, and $v(n)$, respectively. Furthermore, a sequence to be processed will consist of a portion of the speech waveform as viewed through a window $w(n)$, so that

$$s(n) = [p(n) \otimes g(n) \otimes v(n)]w(n). \quad (7)$$

If we assume that the term $g(n)\otimes v(n)$ has an effective duration of N samples, and $w(n)$ is smooth compared with this term so that $w(n_1)\cong w(n_1+N)$ for any n_1, then (7) can be approximated as

$$s(n) \cong [p(n)w(n)] \otimes g(n) \otimes v(n).$$

Let us first consider the term $p(n)w(n)$ which we abbreviate as $p_1(n)=p(n)w(n)$. Treating the sequence of pitch pulses $p(n)$ as a train of equally spaced unit samples with spacing of τ samples, then the z-transform of $p_1(n)$, denoted by $P_1(z)$, is

$$P_1(z) = \sum_{k=-\infty}^{\infty} w(k\tau)(z^{\tau})^{-k}.$$

Let $w_\tau(n)$ be defined as the window compressed in time by τ, so that $w_\tau(n)=w(n\tau)$ with $W_\tau(z)$ and $\hat{w}_\tau(n)$ denoting the z-transform and complex cepstrum, respectively, of $w_\tau(n)$. Then,

$$P_1(z) = W_\tau(z^\tau)$$

and

$$\log P_1(z) = \log W_\tau(z^\tau)$$

so that $\hat{p}_1(n)$, the complex cepstrum of $p_1(n)$, is given by

$$\begin{aligned}\hat{p}_1(n) &= \hat{w}_\tau\left(\frac{n}{\tau}\right) & n &= 0, \pm\tau, \pm 2\tau, \cdots \\ &= 0 & &\text{otherwise.}\end{aligned} \quad (8)$$

[2] As discussed in Noll,[2] there has been a variety of definitions of the cepstrum. In the present context, the cepstrum is considered as the inverse Fourier transform of the log magnitude of the Fourier transform.

In other words, the complex cepstrum of a train of pitch samples weighted with a window can be determined by compressing the window by a factor τ corresponding to the spacing between pitch samples, determining the complex cepstrum, and expanding the result by a factor τ.[3] We observe that $\hat{p}_1(n)$ as expressed by (8) consists of a train of samples with spacing τ, in which the mth sample has a weighting $\hat{w}_\tau(m)$.

If we assume that the original window $w(n)$ is a minimum phase window, i.e., that all poles and zeros of its z-transform lie inside the unit circle, then all poles and zeros of the z-transform of $w_\tau(n)$ lie inside the unit circle, and, hence, $\hat{w}_\tau(n)$ is zero for $n<0$. For a minimum phase window, then, the complex cepstrum of $p_1(n)$ will be zero for $n<0$. Similarly, if $w(n)$ is maximum phase, such that all poles and zeros of its z-transform lie outside the unit circle, then $\hat{p}_1(n)$ will be zero for $n>0$. If the window is symmetric then $\hat{p}_1(n)$ will be symmetric.

The complex cepstrum of the sequence $v(n)$ corresponding to the vocal tract impulse response can be derived, if we assume that it is representable as a cascade of damped resonators. In this case, $v(n)$ is minimum phase and $V(z)$ is of the form

$$V(z) = \frac{K}{\prod_{i=1}^{M}(1-a_i z^{-1})(1-a_i^* z^{-1})} \qquad |a_i| < 1.$$

Since all poles of $V(z)$ are inside the unit circle, $\hat{v}(n)$ is determined from (4), and is given by

$$\hat{v}(n) = -\frac{1}{2\pi j n}\oint\left[z\frac{V'(z)}{V(z)}\right]z^{n-1}dz \qquad n \neq 0.$$

Evaluating the integrand,

$$z\frac{V'(z)}{V(z)} = -\sum_{i=1}^{M}\frac{a_i z^{-1}}{(1-a_i z^{-1})} - \sum_{i=1}^{M}\frac{a_i^* z^{-1}}{(1-a_i^* z^{-1})}$$

and

$$\begin{aligned}\hat{v}(n) &= +\frac{1}{2\pi j n}\sum_{i=1}^{M}\left[\oint\frac{a_i z^{-1}}{1-a_1 z^{-1}}z^{n-1}dz + \oint\frac{a_i^* z^{-1}}{1-a_i^* z^{-1}}z^{n-1}dz\right] \\ &= \frac{1}{n}\sum_{i=1}^{M}[(a_i)^n + (a_i^*)^n] \qquad n > 0 \\ &= 0 \qquad n < 0\end{aligned}$$

or

$$\begin{aligned}\hat{v}(n) &= \sum_{i=1}^{M}\frac{|a_i|^n}{n}\cos\omega_i n \qquad n > 0 \\ &= 0 \qquad n < 0\end{aligned} \quad (9)$$

[3] The derivation of this result assumes that the phase associated with the window is zero at $\omega = \pm\pi$.

where

$$a_i = |a_i| e^{j\omega_i}.$$

An accurate analytical representation of the glottal pulse $g(n)$ is not known and, consequently, it is difficult to make any specific statements regarding $\hat{g}(n)$. If we assume that it is a time-limited pulse, then its z-transform is representable entirely by zeros (with the exception of poles at $z=0$). We can expect, in general, that $G(z)$ will contain zeros both inside and outside the unit circle, in other words will be nonminimum phase.[4] We can express $g(n)$ as the convolution of a minimum phase sequence and a maximum phase sequence, so that

$$g(n) = g_1(n) \otimes g_2(n)$$

where $g_1(n)$ is minimum phase, and $g_2(n)$ is maximum phase. The complex cepstrum of $v_1(n)$ will be zero for $n<0$, and will be dominated for large n by the zeros closest to the unit circle. If e^{σ_1} represents the distance from the origin in the z-plane to the zero of $v_1(n)$ closest to the unit circle, then for large positive n, $\hat{v}_1(n)$ behaves as $e^{n\sigma_1}/n$. Similarly, if e^{σ_2} represents the distance from the origin in the z-plane to the zero of $v_2(n)$ closest to the unit circle, then for large negative n, $\hat{v}_2(n)$ behaves as $e^{n\sigma_2}/n$. Consequently, the duration of the complex cepstrum of the glottal pulse will be governed by the zeros of its z-transform which are closest to the unit circle. We will assume that e^{σ_1} and e^{σ_2} are such that the complex cepstrum of the glottal pulse has an effective duration which is less than a pitch period.

From the above arguments, we can consider dividing the complex cepstrum into three regions. Comparing (8) and (9), the complex cepstrum of the vocal tract decays rapidly relative to the contribution from pitch. Consequently, for the magnitude of n greater than or equal to a pitch period, the primary contribution is due to pitch. For positive values of n less than a pitch period, the contribution is from the vocal tract and the minimum phase component of the glottal pulse. For negative n, the contribution is from the maximum phase component of the glottal pulse.

To recover $p_1(n)$, we wish to keep only those points in $\hat{s}(n)$ for n greater than, or equal to, a pitch period. If the number of points in a pitch period is n_p, then the linear filter corresponds to multiplying $\hat{s}(n)$ by zero for $|n| < n_p$ and by unity for $|n| > n_p$. To recover the term $g_1(n) \otimes v(n)$, we multiply $\hat{s}(n)$ by zero for $n<0$ and $n \geq n_p$, and to recover $g_2(n)$, we retain only those values of $\hat{s}(n)$ for $-n_p < n < 0$. After filtering, the result is transformed by the inverse of the system D.

Since no clear statements can be made about the relative importance of the maximum and minimum phase components of the glottal pulse, the notion of recovering the maximum phase component has no obvious implications. However, if we retain values in the complex cepstrum for small positive and negative values of n, then combined vocal tract and glottal pulse information can be recovered with the appropriate phase relations.

An an example of the recovery of pitch, consider the vowel "ah" as in father, shown in Fig. 3(a). The speech was sampled at 10 kHz and weighted with a Hanning window 25.6 ms in duration. The complex cepstrum is

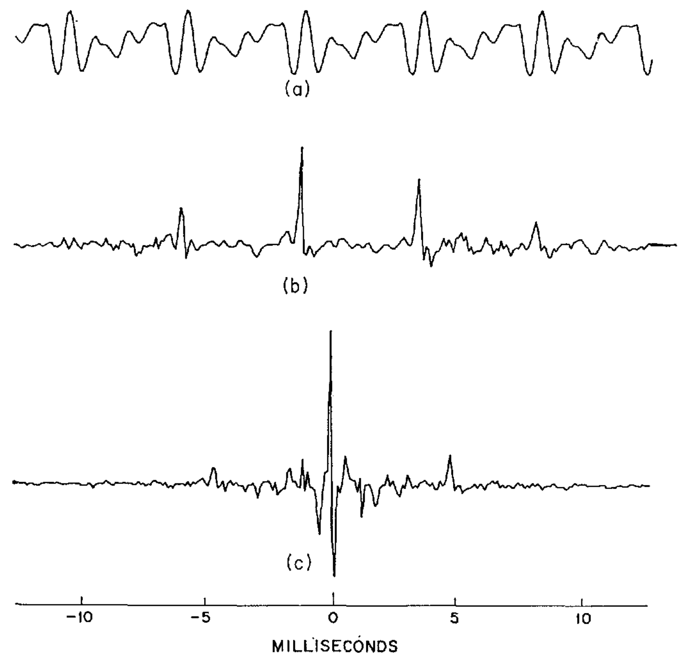

Fig. 3. (a) Sample of the vowel "ah." (b) Resulting output due to pitch. (c) Complex cepstrum of (a).

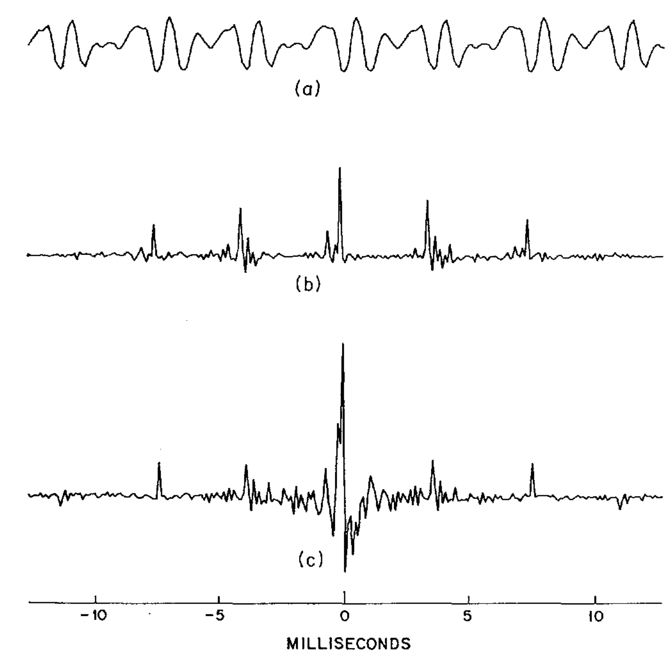

Fig. 4. (a) The speech sample of Fig. 3 resynthesized with alternating pitch. (b) and (c) Output due to pitch and the complex cepstrum, respectively.

[4] Mathews, Miller, and David[4] have argued that the asymptotic zeros of the glottal pulse lie in the right half of the s-plane, if the slope of the leading edge plus the slope of the trailing edge is negative. In such cases, then, the glottal pulse is nonminimum phase.

shown in Fig. 3(c), and the weighted output due to the pitch pulses is shown in Fig. 3(b).

To investigate the method for cases of varying pitch, a single pulse was obtained by "low time filtering" the complex cepstrum to recover the term $v(n) \otimes g(n)$. The speech wave was then resynthesized by convolving this pulse with a train of unit samples with alternating spacing. Fig. 4 shows the resynthesized speech, weighted output due to the pitch pulses, and complex cepstrum for alternating pitch.

From Fig. 3, it appears that for constant pitch there is no particular advantage in carrying out pitch detection on the recovered pitch pulses rather than the complex cepstrum, and in many similar examples tried, the peak is as evident in the cepstrum as in the complex cepstrum. Thus, for examples of this type, where pitch is constant, the inclusion of phase information and processing by the inverse of the system D seems to offer no advantage over cepstral pitch detection, as discussed by Noll. From Fig. 4, however, it appears that this processing places in evidence individual variations between pitch periods. These conclusions, however, are based on a small number of examples, and must be considered as tentative without further experimental verification.

A Method for Inverse Filtering

In the previous discussion, the approach taken was to transform the convolved components of speech into additive components in the complex cepstrum, and attempt to separate them by linear filtering. An alternative approach is to remove the unwanted components by subtracting them, and processing the result by means of the system D^{-1}. This approach is entirely equivalent to processing the original waveform with a linear filter whose frequency response is the reciprocal of the Fourier transform of the unwanted components, i.e., inverse filtering. This approach to filtering requires an accurate representation of the components to be removed.

For the recovery of source information, corresponding to a train of glottal pulses, the signal to be removed is the vocal tract impulse response, or formant structure of the speech. R. Miller[5] and others[6],[7] have successfully used the method of inverse filtering, determining the parameters of the inverse filter by trial and error. In the work by J. Miller, the parameters are determined by matching the speech spectrum. In general, it appears to be difficult to locate the formant frequencies and bandwidths accurately in the presence of the spectral fine structure, due to pitch. In the spectral matching carried out by J. Miller, this fine structure was removed by using single pitch periods.

From the previous discussion, it is clear that the vocal tract and glottal pulse information can be separated from the spectral fine structure or pitch by retaining only those values of the cepstrum for values of $|n|$ less than a pitch period. This corresponds to linear smoothing of the log spectrum to obtain the spectral envelope. Vocal tract parameters can then be obtained by match-

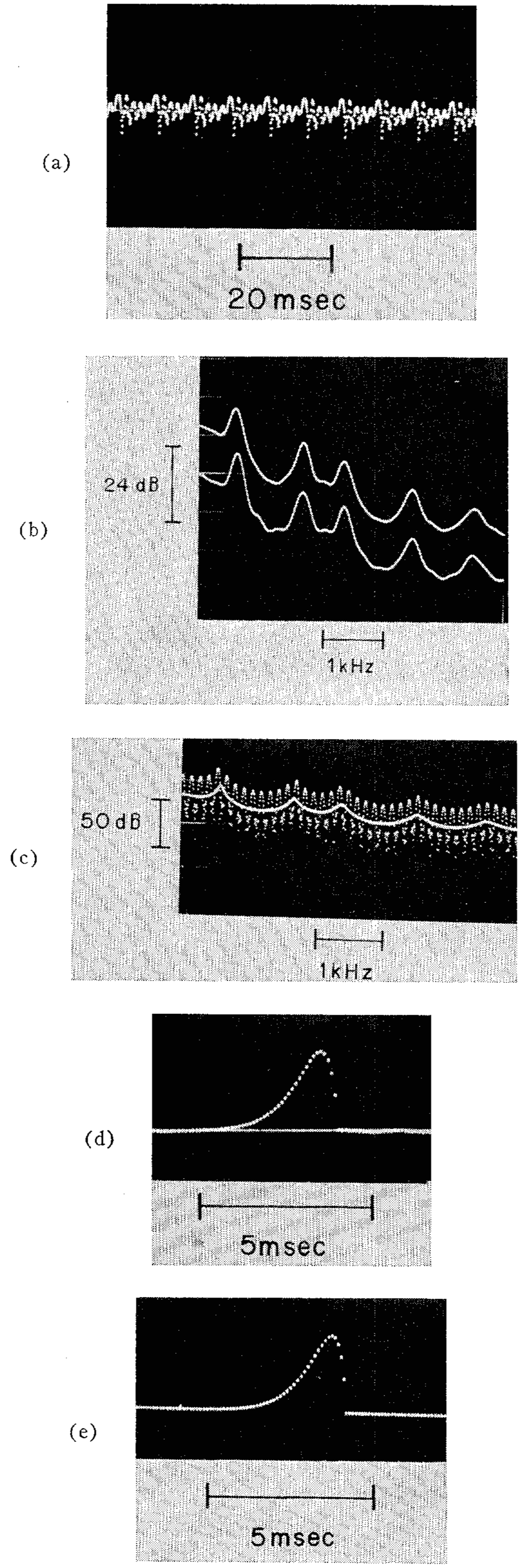

Fig. 5. (a) Synthetic vowel used to illustrate spectral matching and inverse filtering. (b) Lower trace: log spectral envelope of speech sample. Upper trace: smoothed log spectrum of ideal resonators. (c) Log spectrum of speech sample, and unsmoothed log spectrum of resonators superimposed. (d) Recovered "glottal" pulse. (e) Original pulse used to generate the synthetic speech.

ing the smoothed log spectrum with the log spectrum of a set of ideal cascade resonators. Since smoothing of the log spectrum introduces some distortion into the spectral envelope, it is desirable to do the same linear smoothing on the log spectrum of the ideal resonators. Equation (9) specifies an analytic expression for the cepstrum of the resonators, and, therefore, it is a straightforward procedure to generate the cepstrum directly, weighting with the same window used on the cepstrum of the original speech for smoothing of the log spectrum. When a reasonable match has been obtained, the complex cepstrum of the ideal resonators is subtracted from that for the original speech, and the output of the system D^{-1} is determined. To illustrate the procedure, consider the synthetic vowel "ah" of Fig. 5(a), which has been sampled at 10 kHz. To obtain the smoothed log spectrum, the cepstral values for $n<36$ were used. The resulting smoothed log spectrum consists of both vocal tract and glottal pulse information. In Fig. 5(b), the lower trace represents the smoothed log spectrum of the synthetic speech, and the upper trace is the smoothed log spectrum of a set of resonators, together with a double order pole to represent the glottal spectrum (a term to approximate the glottal spectrum is included only to facilitate the matching). In Fig. 5(c), the unsmoothed log spectrum of the original and the matching spectrum are superimposed. The resonator frequencies and bandwidths are identical for Fig. 5(b) and (c). The log spectrum of the glottal pulses are obtained by subtracting the resonator spectrum from the unsmoothed speech spectrum. The "glottal" pulse obtained for this example is shown in Fig. 5(d). For comparison, the original pulse used to generate the synthetic speech is shown in Fig. 5(e).

References

[1] B. P. Bogert, M. Healy, and J. Tukey, "The quefrency alanysis of time series for echoes: cepstrum, pseudo-autocovariance, cross-cepstrum and saphe cracking," in *Proc. Symp. on Time Series Analysis*, M. Rosenblatt, Ed. New York: Wiley, 1963, ch. 15, pp. 209–243.

[2] A. M. Noll, "Cepstrum pitch determination," *J. Acoust. Soc. Am.*, vol. 41, pp. 293–309, February 1967.

[3] A. V. Oppenheim, "Superposition in a class of nonlinear systems," *1964 IEEE Internat'l Conv. Rec.*, pt. 1, pp. 171–177.

[4] M. V. Mathews, J. E. Miller, and E. E. David Jr, "Pitch synchronous analysis of voiced sounds," *J. Acoust. Soc. Am.*, vol. 33, pp. 179–186, February 1961.

[5] R. Miller, "Nature of the vocal cord wave," *J. Acoust. Soc. Am.*, vol. 31, pp. 667–677, June 1959.

[6] J. E. Miller, private communication.

[7] P. B. Carr and D. Trill, "Long-term larynx-excitation spectra," *J. Acoust. Soc. Am.*, vol. 36, pp. 2033–2040, November 1964.

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