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Further Results on Fast Iterative Coding for Feedback Channels: Multiple-Access and Partial-Feedback

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Abstract — **The compressed-error-cancellation framework of [1] is extended for multiple-access channels with feedback and single-user channels with partial feedback.**

I. Introduction

In [1], we develop the compressed-error-cancellation framework for coding for feedback channels and apply it to single-user channels with complete feedback. In this paper, we summarize some recent extensions of the framework for multiple-access channels and channels with partial feedback.

II. Multiple-Access Channels

Consider a discrete memoryless multiple-access channel with feedback with channel transition probability function $q_{Y|X_1,X_2}$ and input probability mass functions q_{X_1} and q_{X_2}. Let X_1, X_2, and Y be distributed such that $p_{Y,X_1,X_2}(y,x_1,x_2) = q_{Y|X_1,X_2}(y|x_1,x_2)q_{X_1}(x_1)q_{X_2}(x_2)$. Then consider the following coding scheme based on the compressed-error-cancellation framework:

User 1 (Tx-1):

- Source 1 produces $N_1 = k_1 n_1$ message bits to be sent in k_1 submessages, each of n_1 bits, to Rx.
- Tx-1 sends the first n_1-bit submessage using the fixed-length variant of the iterative feedback scheme from [1], taking η^{sub} channel inputs, where $n_1/\eta^{\text{sub}} = r_1$, where $r_1 < I(Y;X_1)$.
- Tx-1 sends, in order, each of the second through k_1th n_1-bit submessages using the same fixed-length scheme.

User 2 (Tx-2):

- Source 2 produces N_2 message bits to be sent.
- Tx-2 precodes its N_2 bits into the $N_2' = N_2/H(X_2)$ channel inputs $X_2^{N_2'}$; Tx-2 then puts $D_2 < \eta^{\text{sub}}$ random filler inputs into the channel while it waits for Tx-1 to finish sending whichever submessage is in progress.
- The channel corrupts the transmitted data according to $p_{Y|X_1,X_2}$.
- Rx feeds the corrupted data $Y^{N_2'+D_2}$ back to Tx-2.
- From $Y^{N_2'+D_2}$, Tx-2 determines $X_1^{N_2'}$ (with high probability of success as long as n_1 is sufficiently large).
- Tx-2 compresses $X_2^{N_2'}$ into $N_2' H(X_2|Y,X_1)$ bits, and precodes these bits into $N_2'' = N_2' H(X_2|Y,X_1)/H(X_2)$ channel inputs; again, Tx-2 puts appropriate filler data into the channel.

⋮

- After some fixed number of iterations, final residual data are FEC coded using a repetition code and sent.

Simultaneously:

- Based on feedback information, Tx-1 and Tx-2 each send a length-ν sequence indicating whether they will have to subsequently retransmit their respective blocks of data.

With appropriate choices for the various parameters above, this coding scheme can generate codes with rate-pairs arbitrarily close to the rate pair $(I(X_1;Y), I(X_2;Y|X_1))$ and can be combined with time-sharing to achieve any point in the 2-user feedback-free capacity region. It can be shown that the scheme's error probability decays exponentially with blocklength and that the scheme's complexity is uniformly linear. See [2] for details.

III. Partial-Feedback Channels

To extend the compressed-error-cancellation framework to achieve a reduced feedback rate, we use Slepian-Wolf coding. For example, consider a binary symmetric forward channel with crossover probability ϵ. The Slepian-Wolf coding theorem states that it is possible to provide (nearly error-free) feedback at the rate of $H(\epsilon)$ bits per channel output. It is straightforward to adapt the linear-complexity codes of Spielman [3] for use as linear-complexity Slepian-Wolf codes to achieve a feedback rate of $2H(\sqrt{48\epsilon})$, while still allowing the coding scheme to be capacity-achieving on the forward channel with overall uniform linear complexity.

To reduce the feedback rate even further (i.e., arbitrarily close to zero), we can use a concatenated coding technique. Using an FEC code as inner code, we can essentially transform a BSC with crossover probability arbitrarily close to 0.5 to a BSC with arbitrarily low crossover probability. Using as an outer code our partial-feedback iterative coding scheme, whose feedback rate decreases monotonically with the crossover probability, we arrive at a coding scheme with arbitrarily small feedback rate. The price is a scheme that has only non-uniformly linear complexity.

More detailed descriptions of these techniques, application of the techniques to general DMC's, and connections between these techniques and the ARQ protocol are given in [2].

References

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[2] J. M. Ooi, "A Framework for Low-Complexity Communication over Channels with Feedback," Ph.D Thesis, MIT, 1997.

[3] D. Spielman, "Linear-Time Encodable and Decodable Error-Correcting Codes," IEEE Trans. on IT, pp. 1723–1731, Nov. 1996.

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