# Methods for Noise Cancellation based on the EM Algorithm [1]

*Meir Feder*
Dept. of Electrical Engineering and Computer Science
Research Laboratory of Electronics, Room 36-615
Massachusetts Institute of Technology
Cambridge, MA 02139
and
Department of Ocean Engineering
Woods Hole Oceanographic Institution

*Alan V. Oppenheim*
Dept. of Electrical Engineering and Computer Science
Research Laboratory of Electronics, Room 36-615
Massachusetts Institute of Technology
Cambridge, MA 02139

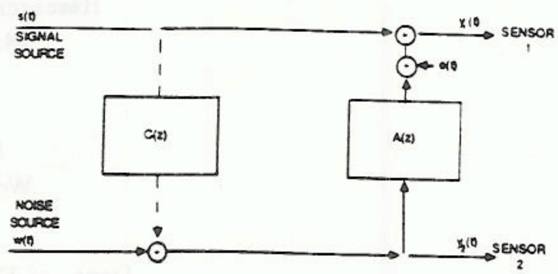
*Ehud Weinstein*
Department of Ocean Engineering
Woods Hole Oceanographic Institution
Woods Hole, MA 02543
and
Department of Electronic Systems
Tel-Aviv University, Tel-Aviv, Israel

[1] This work has been supported in part by the Advanced Research Projects Agency monitored by ONR under contract N00014-81-K-0742 and the National Science Foundation under grant ECS-8407285 at MIT, and in part by ONR under contract N00014-85-K-0272 at WHOI. M. Feder acknowledges the support of Woods Hole Oceanographic Institution.

### Simplified scenario

- **Demonstration 1:**
  $C(z) = 0$. $A(z)$: $10^{th}$ order FIR filter.
  SNR for sensor 1: -20 to -25 dB.

  - *Noisy signal (sensor 1)*
  - *Enhanced signal (EM algorithm, version 1)*
  - *Original (clean) signal*




- **Demonstration 2:**
  $C(z) = 0.1z^{-5}$. $A(z)$: $10^{th}$ order FIR filter.
  SNR for sensor 1: -20 to -25 dB.

  - *Noisy signal (sensor 1)*
  - *Enhanced signal (least-squares method)*
  - *Enhanced signal (EM algorithm, version 1)*
  - *Original (clean) signal*

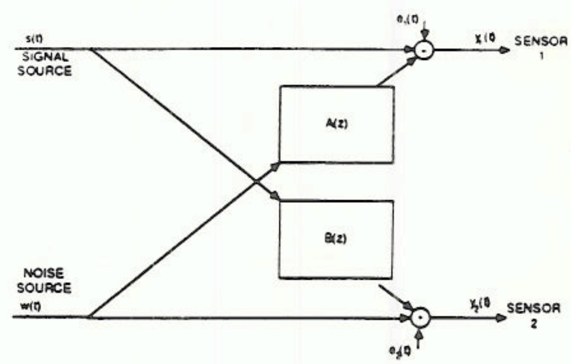
### More general scenario

- **Demonstration 3:**
  $A(z), B(z)$: $10^{th}$ order FIR filters.
  SNR for sensor 1: -20 to -25 dB.

  - *Noisy signal (sensor 1)*
  - *Enhanced signal (least-squares method)*
  - *Enhanced signal (EM algorithm, version 2)*

- **Demonstration 4:**
  $A(z), B(z)$: realistic room impulse responses.
  SNR for sensor 1: +20 dB.

  - *Measured signal (sensor 1)*
  - *Enhanced signal (least-squares method) filter order 250*
  - *Enhanced signal (EM algorithm, version 2) filter order 250*




- **Demonstration 5:**
  $A(z), B(z)$: realistic room impulse responses.
  SNR for sensor 1: 0 dB.

  - *Measured signal (sensor 1)*
  - *Enhanced signal (least-squares method) filter order 500*
  - *Enhanced signal (EM algorithm, version 2) filter order 250*
  - *Enhanced signal (EM algorithm, version 2) filter order 500*

# PROBLEM FORMULATION

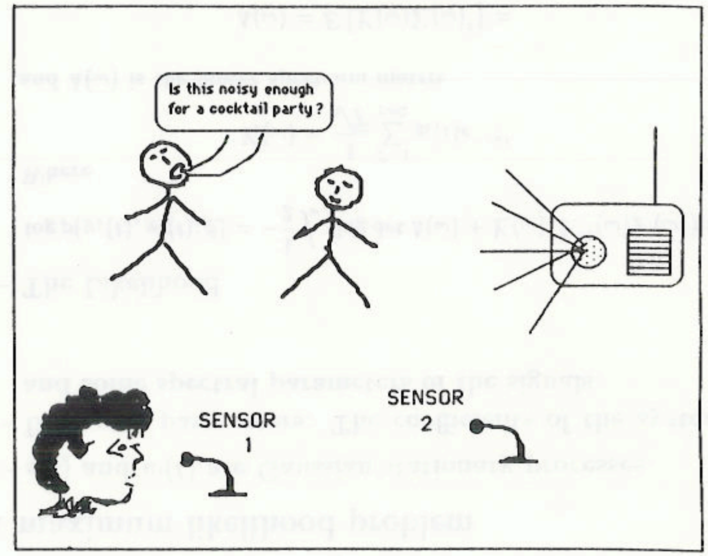
and

# MAIN IDEA

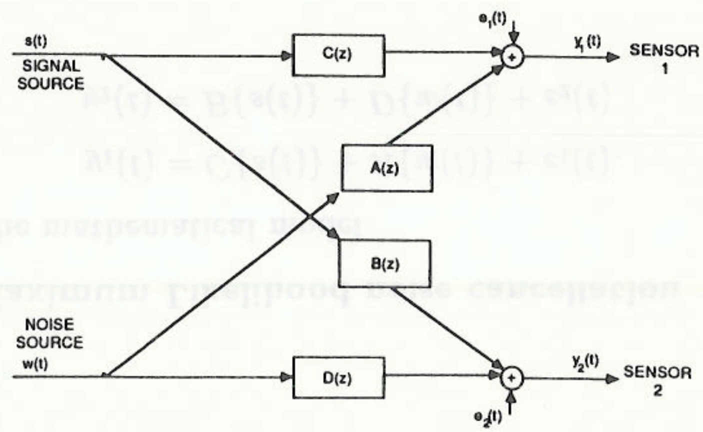
**MAXIMUM LIKELIHOOD APPROACH**

**vs.**

**LEAST SQUARES APPROACH**




The Model:

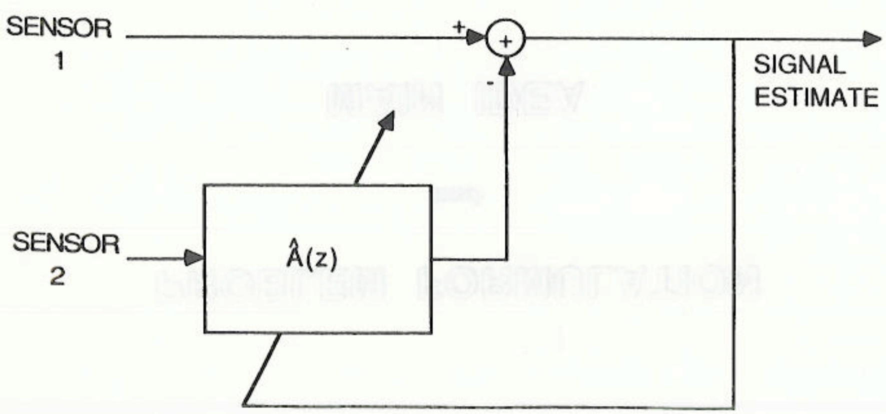


## The least-squares solution (Widrow)

- Estimate the coefficients $\{a_k\}$ of a system $A(z)$, by least-square fitting of $y_2(t)$ (the reference noise signal) to $y_1(t)$ (the primary speech plus noise signal).
- Cancel the noise;

$$\hat{s}(t) = y_1(t) - \sum_{k=0}^{p} \hat{a}_k y_2(t-k)$$

- The least-square solution is approximated by a recursive algorithm; LMS (Widrow), or RLS.




## Maximum Likelihood noise cancellation

- The mathematical model

$$y_1(t) = C\{s(t)\} + A\{w(t)\} + e_1(t)$$
$$y_2(t) = B\{s(t)\} + D\{w(t)\} + e_2(t)$$

- A maximum likelihood problem
  - $s(t)$ and $w(t)$ are Gaussian stationary processes.
  - Unknown parameters: The coefficients of the systems and some spectral parameters of the signals.
  - The Likelihood

$$\log p(y_1(t), y_2(t); \underline{\theta}) = -\frac{1}{2}\int_\omega \left(\log\det\Lambda(\omega) + \underline{Y}(\omega)^\dagger \Lambda^{-1}(\omega)\underline{Y}(\omega)\right) d\omega$$

Where

$$Y_i(\omega) = \frac{1}{\sqrt{T}} \sum_{t=0}^{T-1} y_i(t) e^{-j\omega t}$$

and $\Lambda(\omega)$ is the power spectrum matrix

$$\Lambda(\omega) = E\{\underline{Y}(\omega)\underline{Y}(\omega)^\dagger\} =$$

$$\begin{bmatrix} C(\omega)P_s(\omega)C^*(\omega) + A(\omega)P_w(\omega)A^*(\omega) + \sigma_{e_1}^2 & C(\omega)P_s(\omega)B^*(\omega) + A(\omega)P_w(\omega)D^*(\omega) \\ B(\omega)P_s(\omega)C^*(\omega) + D(\omega)P_w(\omega)A^*(\omega) & B(\omega)P_s(\omega)B^*(\omega) + D(\omega)P_w(\omega)D^*(\omega) + \sigma_{e_2}^2 \end{bmatrix}$$

## Solution, using the EM approach

- Given $A(z)$ and the spectral parameters of $s(t) \implies$ Estimate $s(t)$ and $n(t)$.
- Having $s(t), n(t)$ (and $y_2(t)$) $\implies$
  - Get the maximum likelihood estimate of $\{a_k\}$ and $\sigma^2$ by least-square fitting of $y_2(t)$ to $n(t)$.
  - Get the ML estimate of the spectral parameters of $s(t)$.

### The Algorithm

- **The E step**
  having a current estimate of the parameters
  - Generate a signal $x(t)$

$$x(t) = y_1(t) - \sum_{k=0}^{p} a_k y_2(t-k)$$

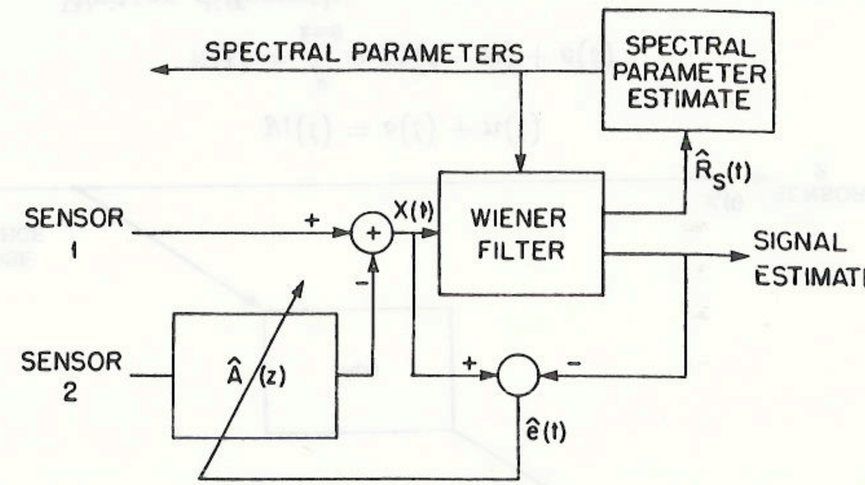
  (Note that given $\{a_k\}$, $x(t) = s(t) + e(t)$.)
  - Apply a Wiener filter to $x(t)$.
  - Get an estimate of $e(t), s(t)$ and of the sample covariance of $s(t)$.
- **The M step**
  - Solve the following least squares problem

$$\min_{\{a_k\}} \sum_t (e(t) - \sum_{k=0}^{p} a_k y_2(t-k))^2$$

  - Add the result to the previous estimate of $\{a_k\}$ to get a new estimate of $\{a_k\}$.
  - Update the signal spectral parameter
    For LPC parameters, solve the normal equation using the estimated sample covariance matrix.

## The suggested processing scheme

*(EM algorithm, version 1)*



MAXIMUM LIKELIHOOD NOISE CANCELLATION

via

THE E-M ALGORITHM

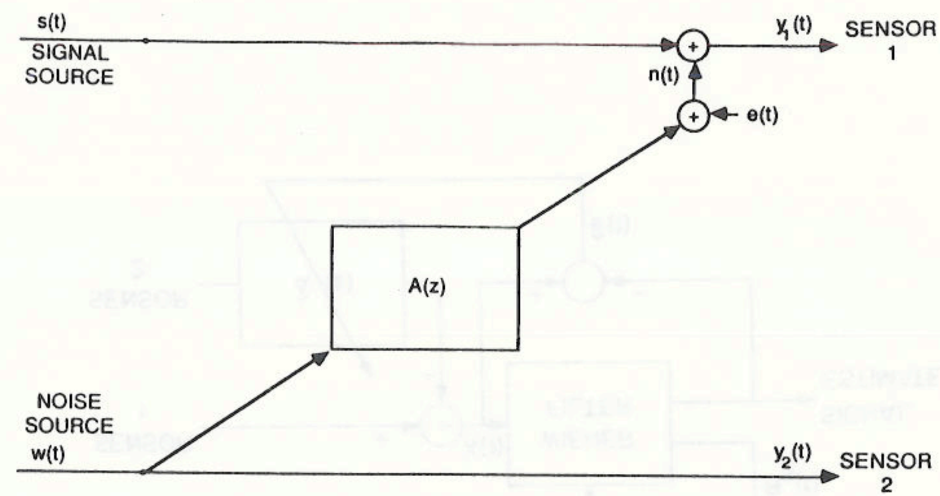
A

SIMPLIFIED

SCENARIO

## Noise cancellation, simplified scenario.

Observe $y_1(t)$ and $y_2(t)$,




$$y_1(t) = s(t) + n(t)$$

$$n(t) = \sum_{k=0}^{p} a_k y_2(t-k) + e(t)$$

Written differently,

$$y_1(t) = s(t) + \sum_{k=0}^{p} a_k w(t-k) + e(t)$$

$$y_2(t) = w(t)$$

MAXIMUM LIKELIHOOD NOISE CANCELLATION

via

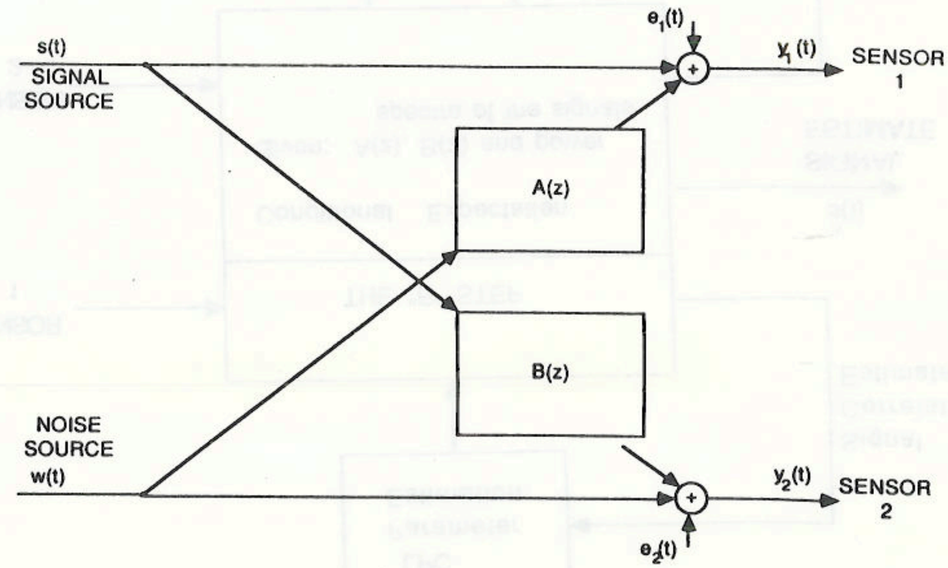
THE E-M ALGORITHM

# A

# MORE GENERAL

# SCENARIO

## Two microphones, more general situation




$$y_1(t) = s(t) + A\{w(t)\} + e_1(t) = s(t) + n_1(t)$$

$$y_2(t) = B\{s(t)\} + w(t) + e_2(t) = w(t) + n_2(t)$$

## Solution, using the EM approach

- The complete data is the outputs *and* inputs of the systems $A(z), B(z)$.
- Given $A(z), B(z)$ and the spectral parameters of $s(t), w(t)$ $\Longrightarrow$ Estimate the complete data and its various quadratic terms.

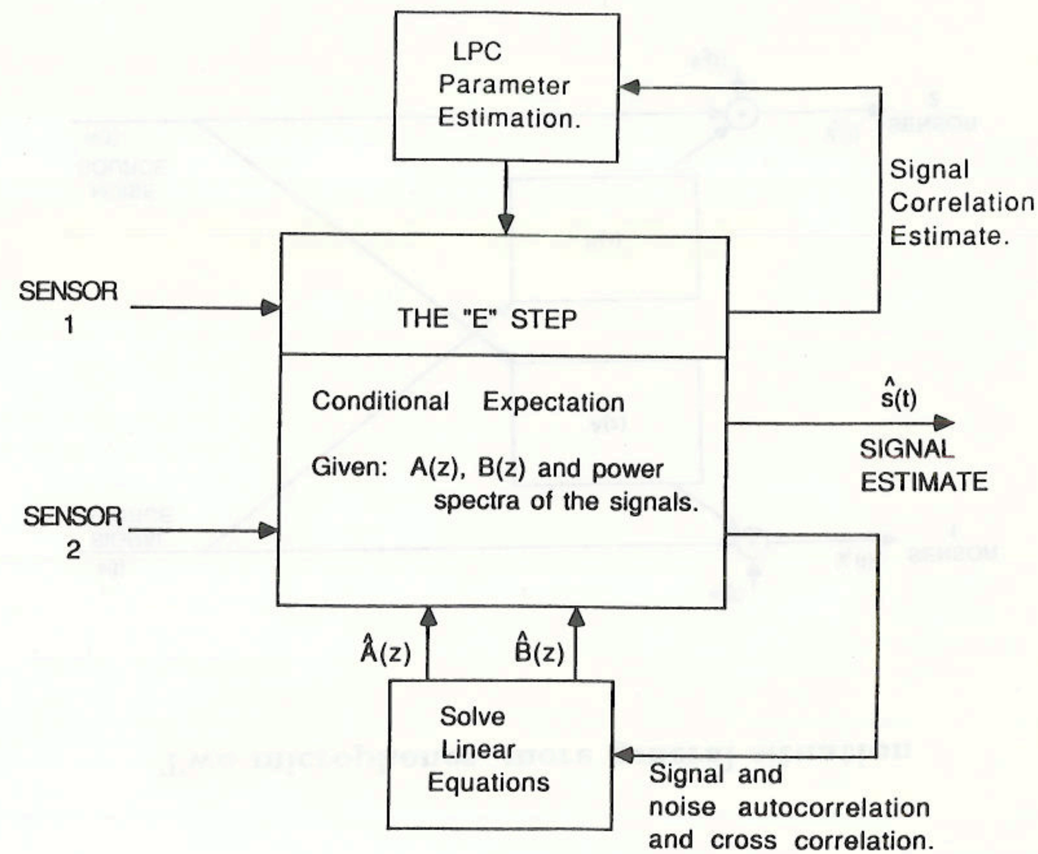
  **(E step)**

- Having the complete data and its quadratic terms $\Longrightarrow$
  - Get the maximum likelihood estimate of $A(z), B(z)$ coefficients by solving linear equations.
  - Get the ML estimate of the spectral parameters.

  **(M step)**

## The suggested processing scheme

*(EM algorithm, version 2)*



## The EM algorithm: Basic theory

- Maximizing the likelihood of the observations $\log p(\underline{y};\underline{\theta})$ *is complicated*
- The observations $(\underline{y})$, can be viewed as "incomplete". The "complete data" $\underline{x}$ contains the observations as:

$$\underline{y} = g(\underline{x}), \qquad g \text{ non-invertible}$$

- If we had observed $\underline{x}$ we would estimate $\underline{\theta}$ easily, i.e

$$\max_{\underline{\theta}} \log p_x(\underline{x};\underline{\theta}) \qquad \textit{is easy !}$$

## The Algorithm

- Start at $\underline{\theta}^{(o)}$
- Iterate:

1. **E step:**

$$Q(\underline{\theta},\underline{\theta}^{(k)}) = E\left\{\log p(\underline{x};\underline{\theta}) \;/\; \underline{y};\underline{\theta}^{(k)}\right\}$$

For exponential family, estimate the sufficient statistics $E\left\{t(\underline{x})/\underline{y};\underline{\theta}^{(k)}\right\}$ and substitute in the expression for the Likelihood of $\underline{x}$.

2. **M step:**

$$\underline{\theta}^{(k+1)} \Leftarrow \max_{\underline{\theta}} Q(\underline{\theta},\underline{\theta}^{(k)})$$

For exponential family, $Q(\underline{\theta},\underline{\theta}^{(k)})$ has the same functional form as $\log p(\underline{x};\underline{\theta})$

## The Estimate Maximize (EM) Algorithm[1]

The Likelihood of the observations is related to the Likelihood of the "complete Data":

$$p(\underline{y};\underline{\theta}) = \frac{p(\underline{x},\underline{y};\underline{\theta})}{p(\underline{x}/\underline{y};\underline{\theta})}$$

But $\underline{y} = g(\underline{x})$ is given. Define the set $\mathcal{X}(\underline{y})$

$$\mathcal{X}(\underline{y}) = \{\underline{x} \mid \underline{y} = g(\underline{x})\}$$

For any $\underline{x} \in \mathcal{X}(\underline{y}) \quad p(\underline{x},\underline{y};\underline{\theta}) = p(\underline{x};\underline{\theta})$

or:

$$p(\underline{y};\underline{\theta}) = \frac{p(\underline{x};\underline{\theta})}{p(\underline{x}/\underline{y};\underline{\theta})}$$

$$\log p(\underline{y};\underline{\theta}) = \log p(\underline{x};\underline{\theta}) - \log p(\underline{x}/\underline{y};\underline{\theta}) \qquad (1)$$

We can substitute in the R.H.S of (1) any $\underline{x} \in \mathcal{X}(\underline{y})$ or we can average it over this set and still have an expression for the likelihood of the observations.

[1] A.P Dempster, N.M Laird and D.B Rubin "Maximum Likelihood from incomplete data via the EM Algorithm", *J. Royal Stat. Soc.* no. 1 pp.1-38, Jan 1977

Suppose we average the R.H.S of (1) using

$$p(\underline{x}/\underline{y};\underline{\theta}') \qquad \underline{\theta}' \; arbitrary$$

$$\underbrace{\log p(\underline{y};\underline{\theta})}_{L(\underline{\theta})} = \underbrace{E\{\log p(\underline{x};\underline{\theta}) \;/\; \underline{y};\underline{\theta}'\}}_{Q(\underline{\theta},\underline{\theta}')} - \underbrace{E\{\log p(\underline{x}/\underline{y};\underline{\theta}) \;/\; \underline{y};\underline{\theta}}_{H(\underline{\theta},\underline{\theta}')}$$

- $H(\underline{\theta},\underline{\theta}')$ is in the form $E_f \log g$
- $H(\underline{\theta}',\underline{\theta}')$ is in the form $E_f \log f \quad f,g$ PDF

**Since** (by Jensen's Inequality)

$$E_f \log f \geq E_f \log g \quad i.e \; H(\underline{\theta}',\underline{\theta}') \geq H(\underline{\theta},\underline{\theta}')$$

**Then if we find** $\underline{\theta}$ **s.t**

$$Q(\underline{\theta},\underline{\theta}') \geq Q(\underline{\theta}',\underline{\theta}')$$

**we have**

$$L(\underline{\theta}) \geq L(\underline{\theta}')$$