

# Fast Iterative Coding for Feedback Channels

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**Abstract** — A class of practical, very low-complexity, variable-rate coding schemes is developed for communication over channels with feedback. It is shown that for arbitrary discrete memoryless channels (DMC's) with noiseless feedback, these schemes achieve error probabilities that decay exponentially with blocklength at any rate below the channel capacity. Extensions of the strategy for use on finite-state channels, known and unknown (universal communication), with feedback are also developed.

## I. INTRODUCTION

Many communication links are bidirectional, supporting the two-way exchange of data. In such scenarios, a natural feedback path exists for each user's transmission. Yet, it is rare that the feedback path is used for anything more than implementing a simple ARQ scheme. That sophisticated feedback codes have not been used stems partly from the lack of a sufficiently rich framework for the design and selection of such codes. In this work, we develop a compressed-error cancellation framework for the design of a class of iterative coding techniques for broad class of channels with feedback.

## II. THE CODING SCHEME

Consider a DMC with feedback (DMC<sub>f</sub>) ( $\mathcal{X}, q_{Y|X}, \mathcal{Y}$ ), where  $\mathcal{X}$  and  $\mathcal{Y}$  are the input and output alphabets respectively, and  $q_{Y|X}$  is the transition probability function. Let  $X^*$  be a random variable such that  $p_{X^*}$  is the capacity-achieving input pmf for the given DMC<sub>f</sub>, and let  $Y^*$  be a random variable such that  $p_{X^*, Y^*}(x, y) = p_{X^*}(x) q_{Y|X}(y|x)$ , for all  $(x, y) \in \mathcal{X} \times \mathcal{Y}$ . Our iterative coding scheme has the following structure [2]:

- The source produces  $N$  message bits to be sent by the transmitter (Tx) to the receiver (Rx).
- Tx precodes and transmits the message bits into  $N' = N/H(X^*)$  channel inputs  $X_1^{N'}$  that look i.i.d. according to  $p_{X^*}$ , where  $H(X^*)$  is the entropy of  $X^*$ .
- The channel corrupts the  $N'$  channel inputs according to  $q_{Y|X}$ .
- Rx feeds the corrupted data  $Y_1^{N'}$  back to Tx.
- Tx uses source coding to compress  $X_1^{N'}$  into  $N'H(X^*|Y^*)$  new data bits.
- Tx precodes the data bits into  $N'' = N'H(X^*|Y^*)/H(X^*)$  channel inputs  $X_{N'+1}^{N'+N''}$ , which look i.i.d. according to  $p_{X^*}$ , and sends them over the channel.
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When the process is continued indefinitely, it follows that the number of channel uses is  $N/I(X^*; Y^*)$ , giving a rate of  $I(X^*; Y^*)$ , the channel capacity. In practice, a finite number of iterations is used, and a final block of data is sent using a modified Schalkwijk-Barron scheme [3].

Ahlsweide considered a similar scheme in [1]. His scheme had fixed block length, its associated error probability did not decay exponentially with blocklength, and practical implementations were not considered. By using arithmetic source coding and a related technique for the precoding, we have developed a practical, variable-length version of this scheme whose error exponent  $E^*(R)$  is at least as great as the modified Schalkwijk-Barron exponent [3]:

$$E^*(R) \geq E_2 \left( 1 - \frac{R}{I(X^*; Y^*)} \right), \quad (1)$$

where  $E_2 = \max_{x \in \mathcal{X}, x' \in \mathcal{X}} \sum_{y \in \mathcal{Y}} q_{Y|X}(y|x) (\log q_{Y|X}(y|x) - \log q_{Y|X}(y|x'))$ . The overall encoding and decoding complexity of the scheme is a remarkable  $O(N)$ .

## III. EXTENSIONS TO FINITE-STATE CHANNELS AND UNIVERSAL COMMUNICATION

This paradigm provides a foundation from which important and practical extensions follow. In particular, the approach has been extended to handle indecomposable finite-state channels. Rates arbitrarily close to

$$H_\infty(\mathcal{X}) + H_\infty(\mathcal{Y}) - H_\infty(\mathcal{X}, \mathcal{Y}) \quad (2)$$

can be achieved, where  $\mathcal{X}$  is a finite-order Markov process chosen by the system designer for the channel input process,  $\mathcal{Y}$  is the resulting channel output process, and  $H_\infty(\cdot)$  is the entropy rate of its argument. We have also extended the approach to unknown indecomposable finite-state channels to create what we refer to as a "universal communication" scheme: for a given input process  $\mathcal{X}$ , the rate approaches (2) where  $\mathcal{Y}$  is the process resulting from the realized channel, which neither the encoder nor decoder knows. For both extensions, we have designed algorithms that allow for encoding and decoding complexities that remain linear in the number of message bits, and error probabilities that decay exponentially, with error exponent bounds analogous to (1). These extensions are developed in detail in [2].

## REFERENCES

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<sup>1</sup>This work has been supported in part by the Advanced Research Projects Agency monitored by ONR under Contract No. N00014-93-1-0686, the Office of Naval Research under Grant No. N00014-96-1-0930, and an AT&T Doctoral Fellowship.