EVENT LOCATION USING RECURSIVE LEAST SQUARES SIGNAL PROCESSING

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ABSTRACT

The problem of locating the position of individual pulses within a group of overlapping pulses can be simplified by preprocessing the data to reduce the overlap. This paper proposes the use of Recursive Least Squares (RLS) prediction for this purpose. The pulse compression performance of two signals derived from the RLS algorithm is compared using all-pole data with different noise levels, and the effects of zeroes in the data and prefiltering are discussed.

I. INTRODUCTION

In many areas including RADAR, SONAR, biomedical and geophysical signal processing, the signal to be processed can be represented as a superposition of pulses with different time delays and the object of the processing is to locate these pulses. When the time durations of the individual arrivals are sufficiently short and the signal-to-noise ratio is high the starting points are easily located either visually or with a simple threshold detector. However, when the S/N is not high and the arrivals overlap, some preprocessing is generally required prior to detection.

In this paper, we propose the use of Recursive Least Squares (RLS) signal modelling and prediction as a preprocessing procedure prior to event detection. RLS generates a time-varying all-pole model of a growing segment of data by recursively updating the model parameters when each new data point becomes available. As we discuss in the next section, both the time varying model coefficients and the prediction error obtained from RLS analysis are potentially useful for event location. These signals contain short duration pulses at the same positions as events in the data and because these short pulses have less overlap they are more easily detected than the original arrivals in the input data.

This work was supported in part by the Advanced Research Projects Agency monitored by ONR under Contract N00014-75-C-0951-NR 049-328 and in part by the National Science Foundation under Grant ENG76-24117.

II. RECURSIVE LEAST SQUARES SIGNAL ANALYSIS

As is well known, modelling data as the response of an all-pole system, and linear predictive analysis are closely related procedures. Two popular methods for performing these types of analysis are the covariance method and the correlation method (1). Both techniques produce a single predictor (or model) which minimizes the total squared prediction error over a region of interest. The difference between them is that the error region to be minimized is infinite for the correlation method and finite for the covariance method. RLS is related to the covariance method and it develops the same predictor for each point in the data that covariance analysis would produce if used on the data up to that point. However, unlike the covariance method which creates but one predictor for a given segment of data and whose error sequence comes from using that one fixed predictor over all the data, RLS dynamically updates its predictor at each new point. Thus, each point in the RLS error is the residual of a time varying predictor which is optimized for an error region up to and including that point (2).

For a single noise free arrival consistent with the all-pole assumption, the RLS prediction error will be non-zero at the first point in the arrival and zero afterwards. With noise present and additional pulse arrivals the signal will no longer be all-pole. However, our experience has shown that RLS still tends to produce a short burst of error and to rapidly change its coefficients during the initial part of each arrival as it adjusts for the change in the data. Consequently, for such data, the RLS prediction error has the potential for time compressing the original arrivals. In addition to the prediction error for event location we have considered a signal derived from the prediction coefficients. Specifically, with the prediction coefficients denoted as $a_i[k]$ where i is the coefficient number and k is the point for which the predictor was optimized, then we define a coefficient change signal C[k] as

$$C[k] = \sqrt{\sum_{i=1}^{k} a_i^2[k] * h[n]}$$

CH1559-4/80/0000-0848$00.75

where h[n] is a single-pole high-pass filter impulse response. This derived signal reacts to changes in the predictor without excessively emphasizing high frequency jitter likely to be present due to noise in the data. If one views the predictor as lying at some point in a-space, then a step change in its position will cause a decaying exponential response in C[k] with a peak height equal to the magnitude of the step.

III. RESULTS

The procedure outlined above were applied to synthetic data generated using the model in Figure 1.

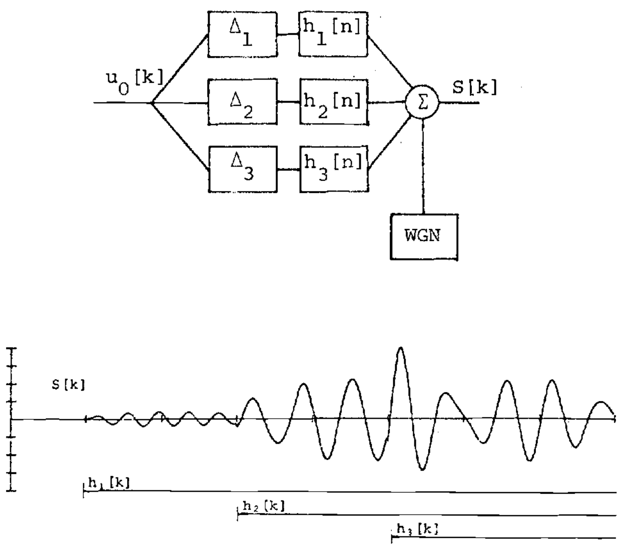

Figure 1: The Data Model

A single impulse is applied to three different paths with separate delays Δ_i and responses $h_i[n]$. These delayed pulses are added to a white Gaussian noise sequence to produce the synthetic signal s[k]. Three arrivals were used in this work because of the physical application of well logging which motivated our interest in the problem. The initial parameters chosen for s[k] were

$$\Delta_1 = 50 \text{ pts} \quad H_1 = \frac{1}{(1-2\cos(.05\pi)z^{-1} + (.98)^2 z^{-2})^2}$$

$$\Delta_2 = 150 \text{ pts} \quad H_2 = \frac{.5}{(1-2\cos(.03\pi)z^{-1} + (.99)^2 z^{-2})^2}$$

$$\Delta_3 = 250 \text{ pts} \quad H_3 = \frac{.2}{(1-2\cos(.04\pi)z^{-1} + (.99)^2 z^{-2})}$$

$$S/N = 50 \text{ db}$$

In Figure 2 is shown the signal, the RLS error and the coefficient change signal. Both location signals mark the start of each arrival well although C[k] appears to be less noisy. Figure 3 shows the same signal with 30db S/N. The noise has totally obscured the first arrival in E[k] and the second is barely visible, whereas three events are more visible in C[k] and the positions of the second and third are accurate. The reason for the emphasis of the noise in the RLS error is that the error is a whitened version of the input signal. The input spectrum has comparatively low amplitude at high frequencies so this whitening takes the general form of a high boost which emphasizes the noise relative to the signal. In contrast, the coefficients tend not to respond to the noise because of its incoherence and therefore C[k] is relatively noise free.

To determine the sensitivity of the two location signals to a small number of zeroes, the system functions in the signal generator were modified as follows:

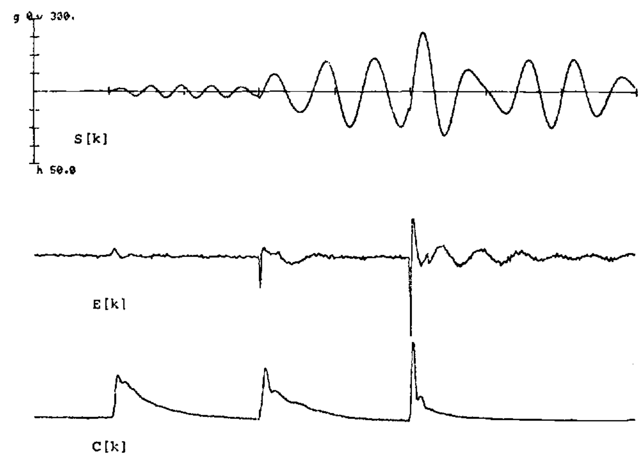

Figure 2: 50 db S/N 12 coefficients

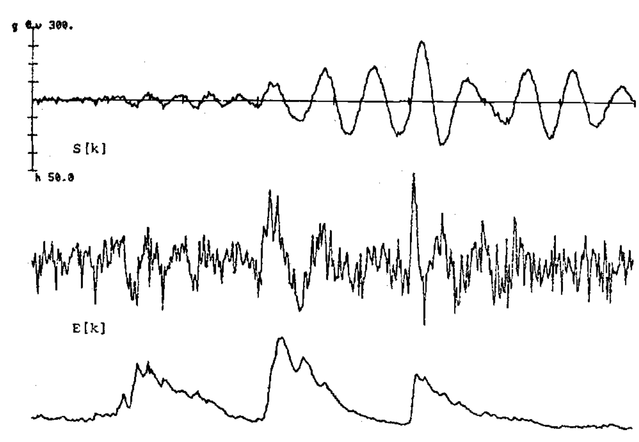

Figure 3: 30 db S/N 12 coefficients

$$H_1 = \frac{1-9z^{-1}}{(1-2\cos(.05\pi)z^{-1} + (.98)^2 z^{-2})^2}$$

$$H_2 = \frac{1-8z^{-1}}{(1-2\cos(.03\pi)z^{-1} + (.99)^2 z^{-2})^2}$$

$$H_3 = \frac{1-5z^{-1}}{(1-2\cos(.04\pi)z^{-1} + (.99)^2 z^{-2})}$$

S/N = 50 db

Figure 4 shows this data and both location signals. As compared with the all-pole case both error location signals produce pulses with slightly longer tails, but the starting location of the events is still clearly visible.

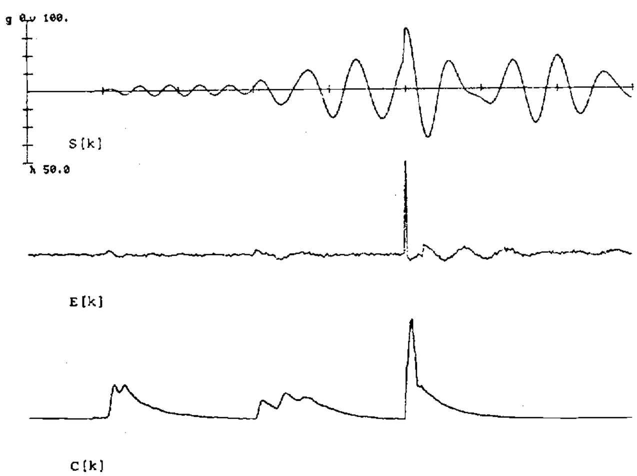

Figure 4: 50 db S/N 12 coefficients real zeroes

IV. CONCLUSIONS

The preliminary conclusion of these investigations is that at high S/N both location signals emphasize the starting point of the arrivals, although C[k] appears to be slightly less "noisy". At low S/N the whitening effect of the RLS error emphasizes the input noise compared to the signal, whereas the coefficient change signal smooths out the noise effects and produces slightly more well defined events.

Finally, we have carried out some preliminary investigations into the use of low-pass prefiltering on the data to reduce noise. Our study centered on whether such filtering degraded the performance of RLS preprocessing, and the results thus far suggest that given the choice of FIR and IIR (all-pole) filters the IIR filter causes the least degradation. We suspect that the reason for this is that FIR filters introduce large numbers of zeroes to the signal and the predictor is unable to work well when the data is not close to being all-pole. In effect the FIR filter introduces many closely spaced arrivals into the signal, while IIR filters only add a few poles to each arrival and therefore IIR filtered data still fits the all-pole model well.

V. REFERENCES

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