

EFFICIENT MULTIUSER DETECTORS FOR INTERSYMBOL INTERFERENCE CHANNELS

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ABSTRACT

A general framework is developed for addressing problems of multiuser detection in wireless communication systems operating in the presence of time- and/or frequency-selective fading due to multipath propagation. In particular, a state-space description of the canonical multiple-access system as a multiple-input multiple-output linear system is used to obtain minimum mean-square error multiuser equalizers having efficient recursive implementations in the form of Kalman filters. These algorithms effectively suppress both intersymbol and interuser (multiple-access) interference, yielding near-far resistant receivers that can reduce power control requirements. More generally, the equalizers make efficient use of all available temporal, spectral, and spatial diversity in the system, and can be used in conjunction with both conventional CDMA and more recently proposed spread-signature CDMA systems. Decision-feedback implementations of the equalizers are also described.

1. INTRODUCTION

In a wide-range of wireless communication applications, there is a need for users to be able to communicate efficiently and asynchronously among themselves in the presence of fading due to multipath propagation. For example, in a cellular mobile radio environment, the transmissions of the individual mobiles pass through generally distinct channels, and a noisy version of their superposition is obtained at the base station.

In such scenarios, obtaining reliable estimates of the symbols transmitted by a particular user (or all users) requires the mitigation of several sources of interference, including multiple-access interference and intersymbol interference.

From a broader perspective, reliable communication in the presence of fading requires that diversity be exploited to improve both average and worst-case performance (outage probabilities). Such diversity takes three forms: spectral, temporal, and spatial. For example, conventional CDMA systems, where all users spread their transmission over a common bandwidth, is a format designed to enable spectral diversity to be exploited. In particular, provided the total bandwidth is large compared to the coherence bandwidth of the channel, then spectral diversity can be exploited by a suitably designed receiver. In this sense, the available diversity manifests itself in the form of intersymbol interference, and the equalizer is the mechanism by which such diversity is actually exploited.

In a similar manner, spread-signature CDMA systems

[1] and spread-response precoding systems [2], in which all users spread the transmission of their symbols in time, are designed to enable temporal diversity to be exploited. In this case, too, a suitably designed equalizer allows this diversity to be exploited. Finally, the use of antenna arrays—either at the receiver [3] or transmitter [4]—in conjunction with suitably designed equalizers allows spatial diversity to be exploited.

There is a substantial literature on the problem of multiple-access interference cancellation in such systems, and a wide range of efficient algorithms have been proposed for use in receivers—see, e.g., [5] [6] [7] [8] [9]. However, in such problems it is generally assumed that the fading in the channel is flat over the total bandwidth and over the symbol duration. By contrast, there have been comparatively few results presented on multiple-access interference cancellation for use on channels without such flat fading. In this work we develop efficient linear minimum mean-square error (MMSE) equalizer structures for such scenarios, which jointly suppress both multiple-access and intersymbol interference and therefore exploit the inherent diversity. In the case, of flat fading, the resulting algorithms yield attractive recursive implementations of some familiar multiuser detection algorithms.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a passband CDMA system in which there are M users, all sharing a total fixed bandwidth LW_0 , so that W_0L/M is the effective bandwidth per user. In the equivalent discrete-time baseband model for the system, the modulation process can be viewed as follows. The coded symbol stream of the m th user ($1 \leq m \leq M$), which we denote by $x_m[n]$, is modulated onto a unique signature sequence $h_m[n]$ to produce $y_m[n]$ which is transmitted within the total available bandwidth.

Conceptually, it is convenient to view the modulation process in two stages. As depicted in Fig. 1, these stages correspond to upsampling (i.e., zero-insertion) by a factor $L \geq M$, followed by linear time-invariant filtering with the signature sequence, i.e.,

$$y_m[n] = \sum_k x_m[k] h_m[n - kL]. \quad (1)$$

We emphasize at the outset that we avoid imposing any constraint on the length K of the signature sequences in this system. In so doing, our results will apply not only to conventional CDMA systems, for which $K = L$, but to recently proposed spread-signature CDMA systems for which $K \gg L$ to allow the temporal diversity benefit to be realized in the presence of time-selective fading.

Often, but certainly not always, the signatures are chosen to satisfy some convenient orthogonal properties. For

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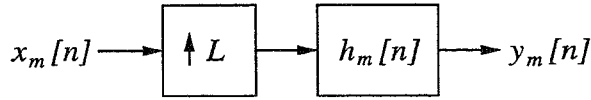


Figure 1. Modulation of the the m th user's coded symbol stream $x_m[n]$ onto a signature sequence $h_m[n]$ for transmission.

example, often the $h_m[n]$ are chosen so that

$$\sum_k h[k - nL] h^T[k - mL] = \delta[n - m] \mathbf{I} \quad (2)$$

where

$$\mathbf{h}[n] = [h_1[n] \quad h_2[n] \quad \cdots \quad h_M[n]]^T. \quad (3)$$

Note that most familiar multiple-access techniques fit into this framework—i.e., in addition to CDMA, both time-division multiple-access (TDMA) and frequency-division multiple-access (FDMA) systems have implementations of the form depicted in Fig. 1 and have signatures satisfying (2). We emphasize, however, that we will *not* assume that (2) is satisfied in our development. In fact, in many cases the users can share the same signature without impacting performance; in such cases, users are distinguished by the channels their respective transmissions pass through. In any event, we will assume that the signature sequences are all known at the receiver.

The multiuser channel we consider, which is depicted in Fig. 2, corresponds to a rather general intersymbol interference environment. In this model, the complex-valued and, in general, time-varying channel response experienced by the m th user's transmission is $a_m[n; k]$, the response at time n to a unit sample input at time k . Hence, the sequence obtained at the receiver is

$$r[n] = \sum_m \sum_k a_m[n; n - k] y_m[n - k] + w[n] \quad (4)$$

where $w[n]$ is a zero-mean, complex-valued stationary circular white Gaussian sequence with variance

$$E [|w[n]|^2] = \mathcal{N}_0. \quad (5)$$

The channel responses $a_m[n; k]$ take into account both the physical propagation medium and path losses (amplitude variations) as well as relative delays among the users transmissions due to the inherent asynchrony in the system. We will frequency restrict our attention to the case in which the $a_m[n; k]$ are causal, so that $a_m[n; k] = 0$ for $k > n$, and finite length, i.e., $a_m[n; k] = 0$ for $k \leq n - N$.

When the channel is time-selective but frequency-nonsselective, corresponding to flat fading (and delays are multiples of the chip time), the $a_m[n; k]$ take the form

$$a_m[n; k] = a_m[n] \delta[n - k - n_m] \quad (6)$$

where n_m is the associated delay. In this case, the received signal (4) specializes to

$$r[n] = \sum_m a_m[n] y_m[n - n_m] + w[n], \quad (7)$$

and only temporal diversity (in the form of, for example, spread-signature CDMA) can be exploited to combat fading.

On the other hand, when the channel is time-nonsselective but frequency-selective, the $a_m[n; n - k]$ are independent of

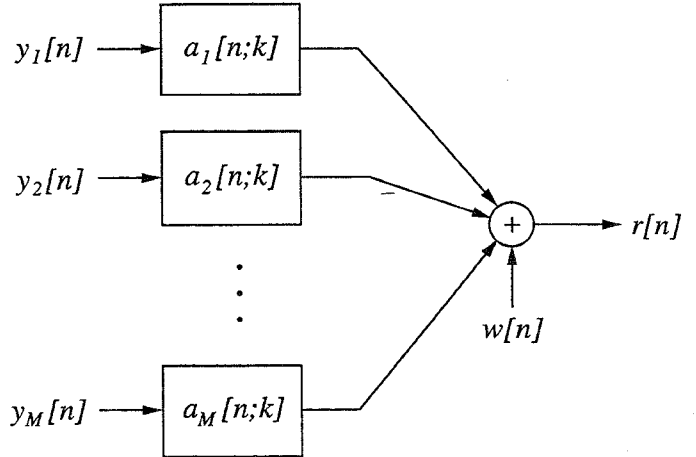


Figure 2. General multiuser intersymbol interference channel model, where $a_m[n]$ denotes the unit-sample response corresponding to the m th user.

n , corresponding to a time-invariant, intersymbol interference channel with constituent unit-sample responses

$$a_m[k] \triangleq a_m[0; -k] = a_m[n; n - k]. \quad (8)$$

In this case, the received signal (4) takes the form

$$r[n] = \sum_m \sum_k a_m[k] y_m[n - k] + w[n], \quad (9)$$

and only spectral diversity can be exploited to combat fading.

In the sequel, it will be convenient to combine the signature modulation process with the effects of the channel to obtain an equivalent model in which the symbol streams of the individual users are time-division multiplexed before transmission over a multiuser channel where the unit-sample responses are now

$$\tilde{a}_m[n; k] = \sum_l a_m[n; l] h_m[l - k], \quad (10)$$

which in the time-invariant case specializes to

$$\tilde{a}_m[n] = a_m[n] * h_m[n]. \quad (11)$$

The multiuser equalization problem we consider is then one of recovering MMSE estimates $\hat{x}_m[n]$ of the symbol streams of the constituent users from a received signal $r[n]$ of the general form (4).

2.1. Polyphase Decompositions and State-Space Models

In developing our equalizers for the multiuser channel, it is convenient to express the received signal as an observation of the state of a multiple-input-multiple-output linear system. Because of the upsampling inherent in the modulation process, the notion of a polyphase decomposition of a signal will be very useful. The L th order polyphase decomposition of an arbitrary signal $p[n]$ is the vector of sequences

$$\mathbf{p}[n] \triangleq [p[nL] \quad p[nL + 1] \quad \cdots \quad p[nL + L - 1]]^T.$$

The polyphase decomposition of a general time-varying channel response $q[n; k]$ is the vector of kernels

$$\mathbf{q}[n; k] \triangleq [q[nL; kL] \quad q[nL + 1; kL] \quad \cdots \quad q[nL + L - 1; kL]]^T. \quad (12)$$

Note that (12) implies that the polyphase decomposition of a specifically time-invariant kernel $q[n]$ has the form

$$\mathbf{q}[n] = [q[nL] \quad q[nL+1] \quad \cdots \quad q[nL+L-1]]^T.$$

With this notation the polyphase decomposition of the received signal can be expressed in terms of the input symbol and the polyphase components of the composite channels

$$\begin{aligned} \mathbf{r}[n] &= \sum_i \sum_k \tilde{\mathbf{a}}_i[n; n-k] x_i[n-k] + \mathbf{w}[n] \\ &= \sum_k \tilde{\mathbf{a}}[n; n-k] \mathbf{x}[n-k] + \mathbf{w}[n] \end{aligned} \quad (13)$$

where

$$\mathbf{x}[n] = [x_0[n] \quad x_1[n] \quad \cdots \quad x_{M-1}[n]]^T$$

is the collection of M symbols transmitted by the collection of users at time n , $\mathbf{w}[n]$ is the polyphase representation for the receiver noise $w[n]$, and

$$\tilde{\mathbf{a}}[n; k] = [\tilde{\mathbf{a}}_0[n; k] \quad \tilde{\mathbf{a}}_1[n; k] \quad \cdots \quad \tilde{\mathbf{a}}_{M-1}[n; k]] \quad (14)$$

is the collection of polyphase decompositions of the channel responses $a_m[n; k]$, $m = 0, 1, \dots, M-1$.

We can write the polyphase decomposition of the received signal as the output of a linear dynamical system of the form

$$\mathbf{s}[n+1] = \mathbf{F} \mathbf{s}[n] + \mathbf{G} \mathbf{x}[n+1] \quad (15a)$$

$$\mathbf{r}[n] = \mathbf{A}[n] \mathbf{s}[n] + \mathbf{w}[n], \quad (15b)$$

where the $\tilde{K}M$ -dimensional state $\mathbf{s}[n]$ is of the form

$$\mathbf{s}[n] = \begin{bmatrix} \mathbf{x}[n] \\ \mathbf{x}[n-1] \\ \vdots \\ \mathbf{x}[n-\tilde{K}+1] \end{bmatrix} \quad (16)$$

with

$$\tilde{K} = \left\lceil \frac{K+N-1}{L} \right\rceil \quad (17)$$

denoting the effective length of the polyphase components of the time-varying channel responses $\tilde{\mathbf{a}}_m[n; k]$. In the state equation (15a), we therefore have

$$\mathbf{F} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix},$$

while in the observation equation (15b) we have that $\mathbf{A}[n]$ is the following $L \times \tilde{K}M$ matrix containing the channel and signature information

$$\mathbf{A}[n] = [\tilde{\mathbf{a}}[n; n] \quad \tilde{\mathbf{a}}[n; n-1] \quad \cdots \quad \tilde{\mathbf{a}}[n; n-\tilde{K}+1]]$$

We assume that the users are uncorrelated and transmit white symbol streams, i.e.,

$$E[\mathbf{x}[n] \mathbf{x}^\dagger[m]] = \mathcal{E}_s \mathbf{I} \delta[n-m].$$

Note that in the time-invariant channel case, the observation matrix $\mathbf{A}[n]$ is a constant \mathbf{A} , i.e., independent of n . We also note that a state space model of the form (15) also applies when in addition there are multiple antennas at the transmitters and receiver in the system. For these extensions, state and/or observation vectors of appropriately higher dimension are used.

3. MMSE MULTIUSER EQUALIZERS

A linear MMSE equalizer for the multiple-access system described by (15) necessarily requires an estimate of the state (16). Such an estimate can be computed recursively and efficiently via the Kalman filtering algorithm. Denoting by $\hat{\mathbf{s}}[n|k]$ the MMSE estimate of the state at time n given observations of $\mathbf{r}[l]$ up to time k —and denoting the associated error covariance by $\Lambda[n|k]$, the state estimation (equalizer) equations take the form [10] [11]

$$\hat{\mathbf{s}}[k|k] = \mathbf{F} \hat{\mathbf{s}}[k-1|k-1] + \boldsymbol{\mu}[k] (\mathbf{r}[k] - \mathbf{A}[k] \mathbf{F} \hat{\mathbf{s}}[k-1|k-1]) \quad (18a)$$

$$\boldsymbol{\mu}[k] = \Lambda[k|k-1] \mathbf{A}^\dagger[k] (\mathbf{A}[k] \Lambda[k|k-1] \mathbf{A}^\dagger[k] + \mathcal{N}_0 \mathbf{I})^{-1} \quad (18b)$$

$$\Lambda[k|k] = (\mathbf{I} - \boldsymbol{\mu}[k] \mathbf{A}[k]) \Lambda[k|k-1] \quad (18c)$$

$$\Lambda[k+1|k] = \mathbf{F} \Lambda[k|k] \mathbf{F}^\dagger + \mathcal{E}_s \mathbf{G} \mathbf{G}^\dagger \quad (18d)$$

and are initialized with $\hat{\mathbf{s}}[-1|-1] = \mathbf{0}$ and $\Lambda[-1|-1] = \mathbf{0}$. Note that although we do not need to compute it explicitly, the one-step prediction is given by

$$\hat{\mathbf{s}}[k+1|k] = \mathbf{F} \hat{\mathbf{s}}[k|k]. \quad (18e)$$

There are several notable features of these estimation equations. First, because of the form of the state vector (16) smoothed estimates of the symbols $\mathbf{x}[n-k]$ for $1 \leq k \leq \tilde{K}-1$ are available at the same time as a filtered estimate of $\mathbf{x}[n]$ is computed. In particular, our estimates of the symbol streams of the users are given by

$$\hat{\mathbf{x}}[n-k|n] = \mathbf{B}[k] \hat{\mathbf{s}}[n|n], \quad 0 \leq k \leq \tilde{K}-1 \quad (19)$$

where

$$\mathbf{B}[k] = [\mathbf{B}_0[k] \quad \mathbf{B}_1[k] \quad \cdots \quad \mathbf{B}_{\tilde{K}-1}[k]] \quad (20)$$

with

$$\mathbf{B}_j[k] = \begin{cases} \mathbf{I} & j = k \\ \mathbf{0} & \text{otherwise} \end{cases} \quad (21)$$

These smoothed estimates are available, in effect, for free—no additional computation is required.

We also remark that further smoothed estimates can be obtained by augmenting the state vector (16) with additional lags of the transmitted symbols, i.e.,

$$\mathbf{s}'[n] = \begin{bmatrix} \mathbf{s}[n] \\ \mathbf{x}[n-\tilde{K}] \\ \mathbf{x}[n-\tilde{K}-1] \\ \vdots \\ \mathbf{x}[n-\tilde{K}'+1] \end{bmatrix} \quad (22)$$

and modifying the state equations (15) by augmenting the matrices \mathbf{F} and $\mathbf{A}[n]$ to obtain \mathbf{F}' and $\mathbf{A}'[n]$, respectively. In practice, computationally more efficient algorithms can be used to obtain these additional smoothed estimates.

Note, too, that the update equation for the state estimate (18a) consists of two parts. The first part is a prediction of the state based on the state estimate at the previous symbol time. The second part is a correction term based on the difference between the prediction and most recently received observation. When the data symbols are discrete-valued (and uncoded), performance can often be enhanced by replacing $\hat{\mathbf{s}}[k-1|k-1]$ with the associated decisions obtained by thresholding. For example, if $x_m[n] \in \{-1, +1\}$, then

$\hat{s}[k-1|k-1]$ can be replaced with $\text{sgn}(\hat{s}[k-1|k-1])$. The result is an attractive decision-feedback equalizer structure.

Finally, note that the algorithm requires that the covariance of the estimation error $\Lambda[k|k]$ be computed at each time step. This reliability information can potentially be used to enhance the performance of a higher level error correction scheme (involving, for example, soft-decision decoding).

Other related equalizer structures can also be related to (18). For example, a recursive implementation of the zero-forcing multiuser equalizer is obtained by replacing the gain (18b) with

$$\boldsymbol{\mu}[k] = \Lambda[k|k-1] \mathbf{A}^\dagger[k] \left(\mathbf{A}[k] \Lambda[k|k-1] \mathbf{A}^\dagger[k] \right)^{-1}. \quad (23)$$

Similarly, a recursive implementation of the matched-filter multiuser equalizer is obtained using the gain

$$\boldsymbol{\mu}[k] = \Lambda[k|k-1] \mathbf{A}^\dagger[k] / \mathcal{N}_0. \quad (24)$$

It is important to emphasize, however, that in these cases the matrices $\Lambda[k|k]$ and $\Lambda[k+1|k]$ no longer correspond to the associated estimation errors, except for limiting cases: $\mathcal{N}_0 \rightarrow 0$ in the zero-forcing case, and $\mathcal{N}_0 \rightarrow \infty$ in the matched-filter case.

3.1. Near-Far Resistance

It is straightforward to verify that given some mild conditions on the channel the MMSE multiuser equalizer is near-far resistant—i.e., that $\Lambda[k|k] \rightarrow \mathbf{0}$ for all $k \geq 0$ when $\mathcal{N}_0 \rightarrow 0$. Note that since $\Lambda[-1|k-1] = \mathbf{0}$ it suffices to show that $\Lambda[k+1|k+1] = \mathbf{0}$ if $\Lambda[k|k] = \mathbf{0}$ and $\mathcal{N}_0 = 0$, and then apply induction. We consider the worst-case scenario corresponding to the maximum number of users, i.e., $M = L$.

From (18d), we obtain $\Lambda[k+1|k+1] = \mathcal{E}_s \mathbf{G} \mathbf{G}^\mathbf{T}$. Hence it suffices to verify that this implies the righthand side of (18c) is $\mathbf{0}$. Letting

$$\mathbf{C}[k] = \mathbf{A}[k] \mathbf{G}, \quad (25)$$

we see that we need only show that

$$\mathbf{C}^\dagger[k] \left(\mathbf{C}[k] \mathbf{C}^\dagger[k] \right)^{-1} \mathbf{C}[k] = \mathbf{I}. \quad (26)$$

However, from (25) we see that $\mathbf{C}[k]$ is the $M \times M$ matrix

$$\mathbf{C}[k] = \tilde{\mathbf{a}}[k; k]. \quad (27)$$

Assuming (27) is invertible for each k then (26) follows immediately. Note that in the time-invariant case, $\mathbf{C}[k]$ is independent of k and takes the form

$$\mathbf{C} = \tilde{\mathbf{a}}[0] = \begin{bmatrix} \tilde{a}_0[0] & \tilde{a}_1[0] & \cdots & \tilde{a}_{M-1}[0] \\ \tilde{a}_0[1] & \tilde{a}_1[1] & \cdots & \tilde{a}_{M-1}[1] \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_0[L-1] & \tilde{a}_1[L-1] & \cdots & \tilde{a}_{M-1}[L-1] \end{bmatrix}. \quad (28)$$

3.2. Steady-State Equalization Error

When the channel is time-invariant and the equalization algorithm is operating in the steady-state, the state update equations reduce to

$$\hat{s}[k] = \mathbf{F} \hat{s}[k-1|k-1] + \boldsymbol{\mu} (\mathbf{r}[k] - \mathbf{A} \mathbf{F} \hat{s}[k-1|k-1]) \quad (29a)$$

$$\boldsymbol{\mu} = \Sigma_\infty \mathbf{A}^\dagger \left(\mathbf{A} \Sigma_\infty \mathbf{A}^\dagger + \mathcal{N}_0 \mathbf{I} \right)^{-1} \quad (29b)$$

where

$$\Sigma_\infty = \lim_{k \rightarrow \infty} \Lambda[k+1|k]$$

is steady-state prediction error covariance. This covariance is obtained as the solution to the Ricatti equation

$$\Sigma_\infty = \mathbf{F} (\Sigma_\infty^{-1} + \mathcal{N}_0^{-1} \mathbf{A}^\dagger \mathbf{A})^{-1} \mathbf{F}^\mathbf{T} + \mathcal{E}_s \mathbf{G} \mathbf{G}^\mathbf{T}, \quad (30)$$

which can be obtained using any of a number of algorithms—see, e.g., [10]. The associated steady-state filtering error covariance

$$\Lambda_\infty = \lim_{k \rightarrow \infty} \Lambda[k|k] = (\mathbf{I} - \boldsymbol{\mu} \mathbf{A}) \Sigma_\infty \quad (31)$$

takes the form

$$\begin{aligned} \Lambda_\infty &= \Sigma_\infty - \Sigma_\infty \mathbf{A}^\dagger \left(\mathbf{A} \Sigma_\infty \mathbf{A}^\dagger + \mathcal{N}_0 \mathbf{I} \right)^{-1} \mathbf{A} \Sigma_\infty \\ &= \mathcal{N}_0 \left(\mathcal{N}_0 \Sigma_\infty^{-1} + \mathbf{A}^\dagger \mathbf{A} \right)^{-1}. \end{aligned} \quad (32)$$

where the second equality in (32) follows from (29b), and the third from an application of the matrix inversion lemma [11]. Expressions for the associated steady-state infinite-interval smoothing error covariance follow from the multi-channel Wiener filtering results presented in [12].

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