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DECONVOLUTION OF SEISMIC DATA USING HOMOMORPHIC FILTERING*

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ABSTRACT

Nonlinear systems which obey a principle of superposition under some operation have been termed homomorphic. A class of homomorphic systems which has found wide application is that involving the filtering of signals that have been combined by convolution. The use of homomorphic systems to deconvolve both seismic reflection and teleseismic data has been proposed and explored by a number of researchers with varying success. The resultant strategies may be globally characterized as a deterministic approach to signal analysis in that no account is made for realistic deviations of the data from the idealized time-invariant seismic models. Several novel results are presented in this paper. A new class of systems, called band-pass homomorphic systems, is discussed, which are matched to the band-pass nature of seismic signals. The implementation of homomorphic systems is improved and made more reliable, through the use of a new phase unwrapping technique. Finally, the concept of short-time homomorphic analysis is introduced and new strategies for wavelet estimation by homomorphic filtering are proposed.

I. INTRODUCTION

An important class of seismic analysis techniques is based on a representation of the seismic signal as a convolution of components, with the basic signal processing task being deconvolution.

There are a number of approaches typically available for carrying out a deconvolution. One of the most common is the use of linear inverse filtering; that is, processing the composite signal through a linear filter, whose frequency response is the reciprocal of the Fourier transform of one of the signal components. Obviously, in order to use inverse filtering such components must be known or estimated. Homomorphic signal processing [1] represents an alternative approach to seismic deconvolution. This class of filtering techniques represents a generalization of linear filtering problems.

The objectives of this paper are first to review the various ways in which homomorphic filtering has been explored for use with seismic data processing; and second, to present a number of new results in homomorphic signal analysis involving not only the theory and implementation of homomorphic systems, but the overall analysis strategy as well. Before doing so, however, we discuss in the next section the basic principles of homomorphic filtering.

II. HOMOMORPHIC SYSTEMS FOR CONVOLUTION

Where we are faced with the problem of filtering signals which have been added, we very often use a linear filter. A principal reason is that linear filters are analytically convenient for dealing with signals that have been added. The principal advantage with linear filtering applied to added signals is that if the behavior of the filter for each of the components separately is known, then the behavior for the sum is the sum of the responses, as a consequence of the principle of superposition.

The notion of homomorphic deconvolution is to apply the same basic reasoning to deconvolution. Towards this end, we consider a class of systems matched to convolution in the same way that linear systems are matched to addition. Then, the class of systems that we consider have the superposition property illustrated in Figure 1. It can be shown that this class of systems can be represented by the cascade of three systems as shown in Figure 2.

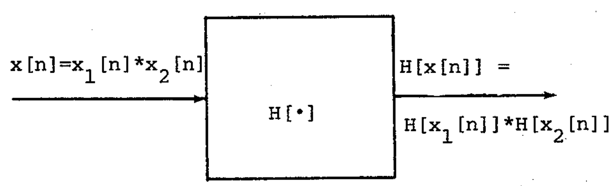

Figure 1: Convolutional Superposition Property

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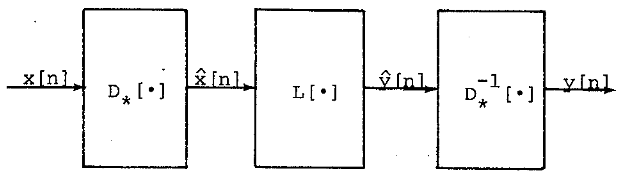


Figure 2: Canonic Representation of Homomorphic Systems for Convolution

The first system, with transformation $D_*[\cdot]$ maps a convolution of components to a sum. The system $L[\cdot]$ is a linear system and the system $D_*^{-1}[\cdot]$ is the inverse of the system $D_*[\cdot]$ and consequently maps a sum of components to a convolution. The system $D_*[\cdot]$ and its inverse are specified by the characteristics of the signal space with which we are dealing. Thus, all the flexibility within this class lies in the linear system $L[\cdot]$.

Full-Band Homomorphic Systems

The transformation $D_*[\cdot]$ has been defined, more specifically, by the property that the z-transform (or Fourier transform) of its output is equal to the complex logarithm of the z-transform (or Fourier transform) of its input, i.e., if:

$$\hat{x}[n] = D_*[x(n)] \tag{1a}$$

then

$$\hat{X}(z) = \log X(z) \tag{1b}$$

Since the z-transform is a complex-valued function its logarithm must be carefully defined. In particular, since the transformation $D_*[\cdot]$ maps convolution to addition, the transformation of eq. (1b) is required to map multiplication to addition and this imposes a particular interpretation on the complex logarithm. The input space to $D_*[\cdot]$ is restricted to the set of all full-band sequences $x[n]$, that is, all stable sequences with stable inverses. Otherwise, the complex logarithm would not be analytic on the unit circle and the output of $D_*[\cdot]$ would be unstable. Then, in particular, low-pass, high-pass and band-pass filtered sequences are not allowable inputs to a homomorphic system with a system $D_*[\cdot]$ as in equation (1). We shall refer to this class of systems as full-band homomorphic systems.

The output $x[n]$ of the system $D_*[\cdot]$ is referred to as the complex cepstrum of the input $x[n]$. This terminology is motivated by the relationship between this transformation and the cepstrum as proposed by Bogert, Healy and Tukey for the detection of echoes. Specifically, the cepstrum of a signal was defined as the power spectrum of the logarithm of the power spectrum. Since the cepstrum was directed toward echo detection rather than a deconvolution, retention of phase information was not important. Thus, it does not utilize phase information and involves the logarithm of real, positive values. By contrast the output of the system $D_*[\cdot]$ is referred to as the complex cepstrum in reference to the fact that it requires the use of the complex logarithm. It is important to note, however, that the complex cepstrum is a real valued sequence. It should be noted also that the cepstrum as defined by Bogert, Healy and Tukey is proportional to the even part of the complex cepstrum.

Properties of the Complex Cepstrum

In carrying out a deconvolution using homomorphic filtering, an appropriate choice for the linear system $L[\cdot]$ must be made. This choice is intimately connected to a number of properties of the complex cepstrum, as summarized below.

Property 1 - The complex cepstrum of a convolution of two (or more) signals is the sum of the individual complex cepstra.

Property 2 - The complex cepstrum of a minimum-phase sequence is zero for $n < 0$ and the complex cepstrum of a maximum-phase sequence is zero for $n > 0$.

Property 3 - The complex cepstrum $\hat{w}[n]$ of a pulse $w[n]$ whose spectrum is smooth tends to be concentrated around low-time values.

Let us denote by $r[n]$ an impulse train, of the form

$$r[n] = \sum_{k=1}^{M} r_k\, \delta[n-n_k] \;,\; n_{k+1} > n_k,\; r_k \neq 0, \forall k$$

where (r_k, n_k) $k=1,\ldots,M$ denote the arrival times and amplitudes associated with each impulse in $r[n]$. Let us define, in general, a periodic impulse train with period $T > 1$, as one for which the interarrival times are multiples of T, that is:

$$n_k = n_{k-1} + \ell_k \cdot T, \quad k=2,\ldots,M, \forall \ell_n > 0$$

then it can be shown that:

Property 4 - The complex cepstrum $r[n]$ of a periodic impulse train $\hat{r}[n]$ is also a periodic impulse train with the same period, that is:

$$\hat{r}[n] = \sum_{n=-\infty}^{\infty} \hat{r}[k]\, \delta[n-kT]$$

Property 5 - Consider a minimum-phase impulse train for which the first interarrival time is T_1, so that

$$n_2 - n_1 = T_1, \quad n_k \text{ arbitrary}, k > 2$$

Then, the complex cepstrum is zero for $0 < n < T_1$.

In general, $\hat{r}[n]$ is non-zero, only at times 0, n_2-n_1, n_3-n_1, ..., n_M-n_1 as well as at all positive linear combinations of these times. A similar result can be derived for maximum-phase impulse trains.

Property 6 - The cepstral structure of a mixed-phase impulse train may be very sensitive to minor changes in the impulse amplitudes. In general, the cepstra of mixed phase impulse trains exhibit a rather elaborate structure, which within the limits imposed by our present understanding of the cepstral mapping, offer no clues regarding the corresponding time structure and vice versa.

This characteristic should be kept in mind, as it plays a major role in shaping strategies for data analysis by homomorphic filtering, to be described in Section IV.

III. SEISMIC DECONVOLUTION BY TIME INVARIANT HOMOMORPHIC ANALYSIS

Seismic signals are often represented in time-invariant form as the convolution of a seismic wavelet w[n], thought to represent the combined source and earth attenuation effects with an impulse train r[n], representing the earth impulse response. A third component p[n] may also have to be included to represent the presence of water reverberations or ghosting mechanisms.

The structure of the complex cepstrum $\hat{s}[n]$ is such that the complex cepstrum of the seismic wavelet $\hat{w}[n]$, that is the reflector series $\hat{r}[n]$ and that of the reverberation train $\hat{p}[n]$, may tend to occupy disjoint time intervals. For example, if the source pulse has a relatively smooth spectrum, $\hat{w}[n]$ will tend to be concentrated near the origin. If the reflector series is minimum-phase then it contributes to the complex cepstrum only for times greater than or equal to the time of the first reflection. For marine seismograms, the reverberation train is a periodic, minimum-phase impulse series, and its complex cepstrum will therefore be right-sided and periodic with the same period.

This tendency for different components to occupy different time intervals in the cepstral domain suggests the possibility of separating these components by gating (i.e. windowing) the cepstrum. A window around the time origin, for example, would tend to retain the contribution from the source pulse. This low-time windowing on the complex cepstrum corresponds to smoothing (i.e. low-pass filtering the low spectrum).

Cepstral gating was first explored for seismic data by Ulrych [2], and was directed towards the deconvolution of teleseismic events. Here, the data was modelled as

$$s[n] = w[n] * r[n] \tag{2}$$

where r[n] is in general a mixed-phase impulse train. In order to render the complex cepstral component associated with the reflector series as simple and predictible as possible, the traces were exponentially weighted as

$$s'[n] = s[n]\,\alpha^n = (w[n]\alpha^n) * (r[n]\alpha^n) = w'[n] * r'[n]. \tag{3}$$

prior to homomorphic deconvolution. This has the effect of radially scaling the z-transform of a sequence. In particular, any zero of R(z) outside the unit-circle in the z-plane can be moved inside the unit circle by exponential weighting, thus rendering the weighted impulse train r'[n], minimum-phase. The weighted seismic wavelet w'[n] was then recovered by low-time gating, with a cutoff frequency which was determined by observation of the theoretical impulse response of the earth at the recording site, which was known beforehand.

Ulrych's approach to teleseismic deconvolution was further pursued by Ulrych et.al. [3], Clayton and Wiggins [4], and Sommerville [5]. These authors were able to estimate very reasonable source pulses for a number of earthquakes and nuclear explosions, using the homomorphic filtering technique. However, they also pointed out a number of problems encountered in such processes. For example, they recognized the computational difficulties of defining the complex logarithm in regions of low-spectral amplitudes. They also questioned the low time assumption for the complex cepstrum of the source pulse since in some instances the source pulse had sharp spectral zeros thus leading to localized but sharp variation in frequency.

The application of homomorphic filtering in seismic reflection studies is discussed first by Buhl, Stoffa and Bryan [6] in the context of shallow-water marine dereverberation and source deconvolution. Here the data was modelled as:

$$s[n] = w[n] * r[n] * p[n] \tag{4}$$

where p[n] denotes a periodic minimum-phase reverberation operator. Exponential weighting was applied to make the reflector series minimum-phase. Letting T_1 denote the two-way travel time within the first earth layer and T_w denote the water column reverberation and assuming

$$NT_w \leq T_1 < (N+1)T_w$$

it follows that high-time gating the complex cepstrum with a cutoff time T_c greater than NT_w, but less than T_1 will deconvolve the seismic source, eliminate the first N multiples in the reverberation pattern and reduce the remaining multiples to at most 1/(N+1) of their original pulse.

A similar procedure was also investigated for deep water dereverberation and source deconvolution by Buttkus [7]. Here, of course, the cepstra of the reverberation train will have high-time characteristics. Dereverberation may then be accomplished by notch gating the cepstrum at the multiples of T_w.

Gallemore [8] has attempted to evaluate the use of homomorphic deconvolution to marine seismic data and to compare its performance to the use of an optimal tapped delay line filter. The criteria for evaluation were the percent of multiple energy removed, percent of reflector distortion and visual improvement of the data when both methods were applied to synthetic seismic data. The indications from the study were that homomorphic deconvolution appears to have greater potential in shallow water applications while the tapped delay line filter appears to be more efficient for deep water data. Buttkus [7] has also discussed the limitations of seismic deconvolution by cepstral gating. Using experimental evidence he concluded that the success of wavelet estimates by low-time gating was largely dependent on the signal to noise ratio of the input data, the degree of overlapping of the cepstrum components of the source wavelet and of the reflectivity function and the choice of the low-time window.

He proposed then to estimate the wavelet by analyzing only a windowed segment within the trace. The procedure involved first the estimation of the arrival times by observation of spikes in the power cepstrum (the power cepstrum is the square of cepstrum of Bogert, Healy and Tukey) followed by notch filtering the segment's complex cepstrum. Implicit in this procedure is again the assumption that the reflector series segment is minimum-phase.

Finally, Otis and Smith [9] attempted to extend Stockham's blind deconvolution technique [10] to seismic processing. The technique does not involve cepstral gating. Here they considered a suite of seismograms, with spatially stationary source pulse and (minimum-phase) spatially non-stationary reflector series. Then the complex cepstrum of each seismic record will be the sum of the cepstrum of the source pulse, which is non-variable plus the cepstrum of the reflector series, which varies from record to record. By averaging the complex cepstra of several records, an estimate of the non-variable functions can be obtained since the variable functions will tend to a mean value. This procedure has been referred to as log spectral averaging or cepstral stacking. The estimated source wavelet is then used to construct a Wiener inverse filter to use as a deconvolution kernel. A similar approach was also used by Clayton and Wiggins [4]. We outlined above the main issues involved in the use of full-band homomorphic systems for the analysis of time-invariant seismic models. In the remainder of this paper we shall outline recent results [11] in homomorphic signal analysis which greatly enhance the potential and the robustness of homomorphic systems in seismic data analysis.

IV. NEW RESULTS IN HOMOMORPHIC SIGNAL ANALYSIS

IV.1 Homomorphic Band-Pass Systems

Seismic signals are often band-pass filtered prior to sampling and recording, to discriminate against additive noise. As pointed out in Section II, band-pass signals are not allowable inputs to a full-band homomorphic system. Thus, it is necessary to redefine the characteristic system $D_*[\cdot]$ so that its input is matched to the band-pass characteristics of seismic data. Such characteristic systems may be defined as a cascade of two systems, as illustrated in Figure 3.

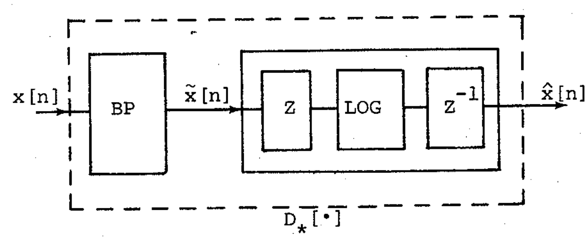


Figure 3: Characteristic System for Homomorphic Band-Pass Filtering

The first system BP corresponds to a frequency scaling operation, that shifts and stretches the pass-band of the signal to occupy the entire band. The second system is then the complex logarithmic system, which is characteristic for full-band homomorphic signal analysis.

Let x[n] be a band-pass signal, satisfying:

$$X[e^{j\omega}] = \begin{cases} X[e^{j\omega}] \neq 0 & w_1 \leq |w| \leq w_2 \\ 0 & \text{elsewhere} \end{cases} \qquad (5)$$

and let $\tilde{x}[n]$ be the response of BP to the input x[n]. Then $\tilde{x}[n]$ satisfies:

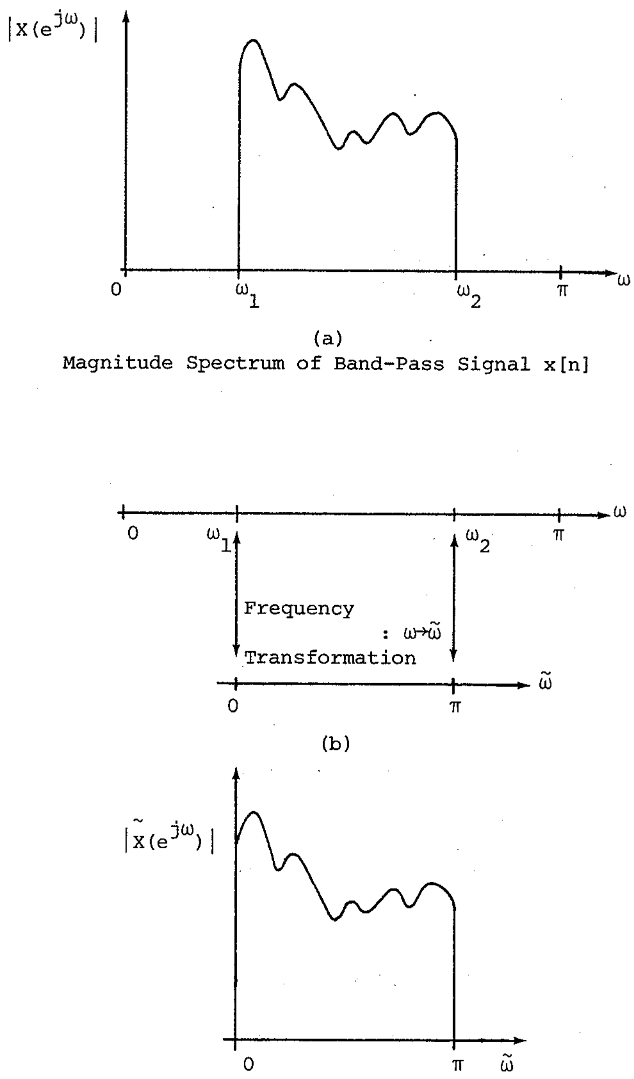
$$\tilde{X}(e^{j\tilde{\omega}}) = X(e^{j\omega}), \quad 0 \leq |\omega| \leq \pi \qquad (6)$$

where

$$\tilde{\omega}(\omega) = \pi \frac{w-w_1}{w_2-w_1} \quad w_1 \leq |w| \leq w_2 \qquad (7)$$

This mapping is illustrated in Figure 4. The system BP is referred to as the band-pass mapping system. Its role in homomorphic filtering corresponds to the role of a linear band-pass filter in linear filtering in that all out-of-band information is effectively discarded.

The class of homomorphic systems with the characteristic systems of Figure 4 is referred to as homomorphic band-pass systems. The theory and implementation of this class of systems can be found in [11].



Magnitude Spectrum of Full-Band Signal $\tilde{x}[n]$

Figure 4: Band-Pass Mapping

IV.2 Phase Unwrapping by Adaptive Integration

The principle computational step in using homomorphic filtering is the computation of the complex cepstrum. The procedure for determination of the complex cepstrum generally relies on an explicit computation of the inverse Fourier transforms of the logarithm of the Fourier transform of the band-pass mapped inputs. This is most easily carried out using the fast Fourier transform algorithm to compute samples of the Fourier transform equally spaced in frequency. Since the Fourier transform is a complex-valued function is logarithm must be appropriately defined. In particular, the phase of a complex number is only unique to within an integer multiple of 2π and this ambiguity must be resolved in computing the phase of the Fourier transform. In order that the logarithmic transformation map multiplication to addition as required, it can be shown that the phase must be generated as a continuous function of frequency. Thus, if the phase is computed modulo 2π, as is typically the case, it must be "unwrapped" to obtain a continuous phase curve.

Simple algorithms, based either on examination of the sampled phase to locate jumps of 2π or based on numerical integration of the phase derivative have proven to be generally unrealiable. As a result, a novel adaptive numerical integration phase unwrapping technique has been proposed [12] which has proven to be very reliable.

IV.3 Determinism and Time-Invariant Homomorphic Analysis

An important new result in homomorphic signal analysis deals with the use of exponential weighting as a means of insuring the minimum-phase character of the reflector series. This practice, which has been used by many researchers as discussed in Section III, is valid in noiseless situations but must be questioned otherwise. As it is shown in [11], by making the reflector series minimum-phase and low-time gating the cepstra, one is in effect constraining the pulse estimate to be a replica of the first arrival. Thus, although this strategy may be reasonable in teleseismic analysis, it is often not appropriate in reflection seismology since the seismic wavelet characteristics change in time, due to attenuation within the earth. Furthermore, the data often contains significant noise components.

In summary, the use of homomorphic systems in seismic data processing has been characterized by a purely deterministic approach to signal analysis, in that no account is made for realistic deviation of the data from the idealized time-invariant models. Such philosophy, on which the use of exponential weighting is based, leads typically to results where the first arrival is perfectly resolved as a very sharp impulse--thus indirectly confirming the matching of the wavelet estimate to the first arrival--the remainder being corrupted by noise, with a severity that depends on the particular time-varying nature of the data and the signal-to-noise ratio.

IV.4 Short-Time Homomorphic Analysis

The homomorphic analysis of signals which approximately follow a convolutional model on a short-time basis is referred to as short-time homomorphic analysis. Here, the basic signal models are of the form:

$$s[n] = [w[n] * r[n]]\, v[n] \qquad (8)$$

where v[n] represents a short-time window. A first attempt to model short-time windowing effects in the context of homomorphic signal analysis was recently reported [13]. In general, we may model a short-time segment as

$$s[n] = w'[n] * r'[n] \qquad (9)$$

where r'[n] represents the windowed reflector segment

$$r'[n] = r[n] \cdot v[n] \qquad (10)$$

and w'[n] is defined in the frequency domain as

$$W'(e^{j\omega}) = S'(e^{j\omega})/R'(e^{j\omega}) \qquad (11)$$

Note that w'[n] depends not only on w[n] but also on the structure of r[n] and on the window shape, onset and duration. If in carrying out short-time homomorphic analysis, an accurate estimate of the seismic pulse w[n] is desired, then one must choose the window v[n] very carefully. In particular, one should not use windows such that r'[n] becomes minimum-phase (or maximum-phase) since then w'[n] will match the onset (or offset) of the segment s[n], which will not usually coincide with a wavelet arrival. By choosing windows which have gradual onset and offset transitions, which are long with respect to the wavelet length and smooth with respect to the reflector series, one minimizes the distortion of the underlying convolutional model by the windowing operation. In that case, one may approximately model s[n] as:

$$s[n] \simeq w[n] * r'[n] \qquad (12)$$

A number of short-time windows have been investigated [11] in the context of short-time homomorphic analysis, namely Hamming, Gaussian, Rayleigh, Linear and Exponential. It was found that in general, the low-time cepstral components of w'[n] and w[n] were essentially identical, as long as the windowed reflector series r'[n] was kept mixed-phase.

V. A STRATEGY FOR SHORT-TIME WAVELET ESTIMATION

Based on the above results, a strategy for wavelet estimation by short-time homomorphic analysis was devised, which capitalizes on the sensitivity of the cepstral structure of mixed-phase aperiodic impulse trains to time domain amplitude perturbations. The procedure consists essentially in using different short-time windows on the same seismic segment. Letting $v_k[n]$, k=1,...,M denote a set of M different short-time windows all defined on the same time-interval, and letting $s_k[n]$ denote the corresponding short-time segments:

$$s_k[n] = [w[n] * r[n]] \cdot v_k[n] \qquad (13)$$

It follows then that, if the windows are chosen as described in Section IV.4:

$$s_k[n] = w[n] * r'_k[n] \qquad (14)$$

where $r'_k[n]$ denotes the windowed reflector segment:

$$r'_k[n] = r[n] \cdot v_k[n] \qquad (15)$$

The complex cepstra of each segment is then of the form:

$$\hat{s}_k[n] = \hat{w}[n] + \hat{r}'_k[n] \qquad (16)$$

where it is expected that the cepstral structure of every impulse train $\hat{r}'_k[n]$ is essentially independent from every other. The seismic wavelet might now be estimated using averaging methods, either in the cepstral or in the time domain.

CEPSTRAL STACKING: Here the wavelet estimate $w_e[n]$ is formed by averaging all the complex cepstra $\hat{s}_k[n]$:

$$\hat{w}_e[n] = \frac{1}{M}\sum_{k=1}^{M} \hat{s}_k[n] = \hat{w}[n] + \frac{1}{M}\sum_{n=1}^{M} \hat{r}'_k[n] \qquad (17)$$

This procedure will give increasingly better results as $M\to\infty$. Thus it may be applied in a fully automatic scheme involving multi-channel/multi-windowing and, within the limits of spatial source stationarity, also multi-shot short-time wavelet estimation, since then the total number of elements averaged may be made quite large.

TIME STACKING: Here, each individual cepstra is low-time windowed, and mapped back to the time-domain by $D_*^{-1}(\cdot)$, that is we compute:

$$s_k^L[n] = D_*^{-1}[\hat{s}_k[n] \cdot \ell[n]] \qquad (18)$$

where $\ell[n]$ denotes a low-time window in the cepstral domain. Since, in general, the cepstrum of each impulse train, $\hat{r}'_k[n]$ will have different contributions in the low-time interval, it follows that:

$$s_k^L[n] \simeq w[n] * r'^L_k[n] \qquad (19)$$

where we defined $r'^L_k[n]$ to be:

$$r'^L_k[n] = D_*^{-1}[\hat{r}'_k[n]\ \ell[n]] \qquad (20)$$

Each low-time component $r'^L_k[n]$ can be essentially modelled as:

$$r'^L_k[n] = \delta[n-m_k] + \eta_k[n] \qquad (21)$$

that is, it consists of an impulse plus a noise component, $\eta_k[n]$. We have observed [11] that the cepstral sensitivity of a mixed-phase impulse train to time-domain amplitude changes is mapped, through the low-time filtering operation, into a similar sensitivity regarding the structure of the noise component $\eta_k[n]$. In other words, it seems reasonable to model the noise component at every instant n as a zero mean random variable with unknown probability distribution. Thus, the wavelet estimate $w_e[n]$ is formed by stacking all the low-time components, after appropriate time-domain synchronization, that is:

$$w_e[n] = \frac{1}{M} \sum_{k=1}^{M} s_k^L[n+m_k] = w[n] * [\delta[n] + \frac{1}{M} \sum_{k=1}^{M} \eta_k[n+m_k]] \quad (22)$$

This technique has been used in the short-time analysis of single-trace noisy synthetic seismograms and has yielded good results. Preliminary evaluation shows that by monitoring this technique the quality of the wavelet estimates is superior to that achieved by cepstral stacking, for the same number of components M.

VI. THE ROLE OF HOMOMORPHIC FILTERING IN REFLECTION SEISMIC DATA ANALYSIS

Seismic analysis by homomorphic filtering is a powerful and promising technique in that the seismic wavelet may be recovered without any a priori assumptions regarding the structure of the wavelet or of the reflection series. In particular, the seismic wavelet is not assumed minimum phase as it happens with zero lag predictive deconvolution, nor the reflector series is assumed uncorrelated as it happens with Wiener filtering techniques. The homomorphic signal analysis strategy must take into account both the signal models and the analysis goals. In particular, the band-pass nature of seismic data--that is, the fact that the signal's spectrum can be modelled as the product of two components only within a given pass-band, may be neatly handled by the class of homomorphic band-pass systems discussed in this paper. The use of homomorphic analysis based on time-invariant signal models may be appropriate in many instances, for example, in shallow water marine dereverberation, and in teleseismic data analysis. Otherwise, the time-varying nature of the data as well as the presence of in-band noise may be handled by analyzing short-time windowed segments and recovering the seismic wavelet which best represents the seismic wavefront for a given suite of reflections. Both cepstral stacking or time-stacking can be used for this purpose, either in an automatic scheme or in a monitoring mode.

The recovery of the reflector series, within each short-time interval may be handled in various ways. For example, Optimum Lag Wiener Spiking filters [14] may be designed, from the seismic wavelet estimate. Alternatively, Homomorphic Predictive schemes [15] , based on the representation of the wavelet estimate by its minimum-phase and maximum-phase components and the synthesis of the deconvolution filter by parameteric linear predictive modelling of these wavelet components has proven to be a very versatile technique, often yielding improved results relative to Optimum Lag Wiener Filtering [11].

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[15] J.Tribolet, A.Oppenheim, G. Kopec, "Deconvolution by Homomorphic Prediction," 45th Annual Int. Meeting of SEG, Oct. 12-16, 1975.

REFERENCES

[1] A. Oppenheim and R. Schafer, Digital Signal Processing, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1975.

[2] T. Ulrych, "Application of Homomorphic Deconvolution to Seismology," Geophysics, vol. 36, no. 4, pp. 650-660, August 1971.

[3] T. Ulrych, O. G. Jensen, R. M. Ellis and P. G. Sommerville, "Homomorphic Deconvolution of Some Teleseismic Events," Bull. of the Seismological Soc. of Am., vol. 62, no. 5, pp. 1253-1265, March 1972.

[4] R. W. Clayton and R. A. Wiggins, "Source Shape Estimation and Deconvolution of Teleseismic Bodywaves," submitted to Geophys.

[5] P. G. Somerville, "Time Domain Determination of Earthquate Fault Parameters from Short-Period P-Waves," Ph.D. Thesis, Dept. of Geophysics and Astronomy, The Univ. of British Columbia, December 1975.

[6] P. L. Stoffa, P. Buhl and G. M. Bryan, "The Application of Homomorphic Deconvolution to Shallow-Water Marine Seismology-Part I: Models and Part II: Real Data," Geophysics, vol. 39, no. 4, pp. 401-426, August 1974.

[7] B. Buttkus, "Homomorphic Filtering, Theory and Practice," Geophysical Prospecting, vol. 23, no. 4, p. 712, 1975.

[8] J. B. Gallemore, "A Comparative Evaluation of Two Acoustic Signal Dereverberation Techniques," S.M. and E.E. Thesis, Mass. Inst. of Tech., Dept. of Elec. Eng. and Comp. Sci., June 1976.

[9] R. B. Smith and R. M. Otis, "Homomorphic Deconvolution by Log Spectral Averaging," submitted to Geophysics.

[10] T.G. Stockham, Jr., T.M. Cannon and R.B. Ingebretsen, "Blind Deconvolution through Digital Signal Processing," IEEE Proceedings, vol. 63, no. 4, pp. 678-692, April 1975.

[11] J.M. Tribolet, "Seismic Applications of Homomorphic Signal Processing," Sc.D. Thesis, Dept. of Elec. Eng. and Comp. Sci., M.I.T., June 1977.

[12] J.Tribolet, "A New Phase Unwrapping Algorithm" IEEE Trans. ASSP, vol. 25, no. 2, April 1977

[13] J.Tribolet, T. Quatieri, A.Oppenheim, "Short-Time Homomorphic Analysis," 1977 IEEE Conf. on ASSP, May 9-11, 1977, Hartford, Conn.

[14] S.Treitel and E.Robinson, "The Design of High-Resolution Digital Filters," IEEE Trans. Geo. Elec., vol. GE-4,no.1, June 1966.