CHANNEL EQUALIZATION FOR SELF-SYNCHRONIZING CHAOTIC SYSTEMS

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ABSTRACT

Most strategies proposed for utilizing chaotic signals for communications exploit the self-synchronization property of a class of chaotic systems. Any realistic communication channel will introduce distortion including time-dependent fading, dispersion, and modification of the frequency content due to channel filtering and multipath effects. All of these distortions will affect the ability of the chaotic receiver to properly synchronize. This paper develops and illustrates some specific approaches to channel equalization to compensate for these distortions for self-synchronizing chaotic systems. The approaches specifically exploit the properties of chaotic drive signals and the self-synchronization properties of the receiver.

1. INTRODUCTION

Over the past several years there has been considerable interest in utilizing chaotic signals for communications. Most strategies that have been proposed exploit the self-synchronization property of a class of chaotic systems [1]-[9]. By necessity, synchronization of the receiver requires that the received drive signal be undistorted or that it first be appropriately equalized in amplitude, spectral content and phase. Specifically, it can be anticipated that a realistic transmission channel will introduce a time varying attenuation due to fading, scattering, etc., will modify the spectral characteristics of the transmitted signal due to channel filtering and multipath and will introduce additive noise. The effects of additive noise on synchronization have been discussed in [10]. In this paper we propose specific techniques for estimating and compensating for the effects of the channel on amplitude and spectral content of the synchronizing drive signal.

An approach to equalizing channels which apply a slowly time-varying attenuation to the transmitted chaotic drive signal was considered in [11] and is illustrated in section 3 of this paper. In this paper we build on those ideas by developing an approach which can be applied to a much broader range of realistic channels. By using the ratio of the power spectra for the transmitted and received drive signals, coarse channel equalization can be obtained. The resulting channel equalizer can then be used as a starting point in the determination of an optimal channel equalizer that minimizes the receiver's synchronization error.

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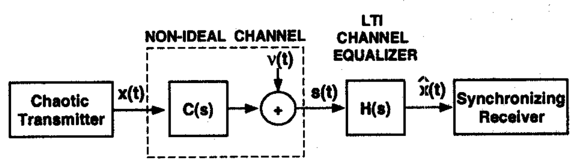

Figure 1. Chaotic communications over non-ideal channels.

2. CHANNEL EQUALIZATION CONCEPTS

In figure 1, we illustrate a scenario in which a chaotic drive signal, $x(t)$, is transmitted over a non-ideal communication channel. The deterministic part of the channel is modeled by an unknown LTI system with transfer function $C(s)$, while the statistical part of the channel is modeled by an additive white noise source, $v(t)$. At the receiver, the corrupted drive signal, $s(t)$, is processed with an LTI equalizer such that $\hat{x}(t) \approx x(t)$. If the equalization is sufficient then the receiver will approximately synchronize to the transmitter.

In practical applications involving self-synchronizing chaotic systems, detailed information regarding $x(t)$ and $C(s)$ would, most likely, not be available at the receiver. However, while $x(t)$ is not known exactly, it's power spectrum, $P_{xx}(j\omega)$, is typically known and can be used for coarse estimation of the frequency response of the equalizer. A channel equalization approach which utilizes the available power spectra information is briefly described below.

2.1. Coarse Channel Equalization

The power spectrum of the received drive signal, $P_{ss}(j\omega)$, is given by

$$P_{ss}(j\omega) = P_{xx}(j\omega)|C(j\omega)|^2 + P_{vv}(j\omega) \quad , \qquad (1)$$

where $P_{vv}(j\omega)$ denotes the power spectrum of $v(t)$. The power spectrum of the compensated drive signal is given by

$$P_{\hat{x}\hat{x}}(j\omega) = P_{ss}(j\omega)|H(j\omega)|^2 \quad . \qquad (2)$$

Consequently, an appropriate choice for $|H(j\omega)|^2$ is given by

$$|H(j\omega)|^2 = \frac{P_{xx}(j\omega)}{P_{ss}(j\omega)} \quad . \qquad (3)$$

With this choice, the power spectrum of the compensated drive signal matches the power spectrum of the transmitted drive signal.

If the channel is assumed to be minimum phase, then $H(j\omega)$ can be constructed from $|H(j\omega)|^2$ using a variety of techniques such as spectral factorization, the complex cepstrum, or the Hilbert transform. If the channel is not minimum phase, however, then this approach may produce a filter that does not provide adequate channel compensation. For example, the spectral ratio approach is certain to fail for all-pass channels. Another drawback of the spectral ratio approach is that the estimate of $|H(j\omega)|^2$ in equation (3) becomes highly ill-conditioned due to the gradual decrease of $P_{xx}(j\omega)$ and $P_{ss}(j\omega)$ at high frequencies. A modified approach incorporates the synchronizing properties of the receiver into the filter design process and is further described below.

2.2. Optimal Channel Equalization

In most cases, perfect channel compensation is not possible. Since our objective is to improve the synchronization between transmitter and receiver, we choose as our criterion minimization of synchronization error.

In developing the optimal channel equalizer we assume that the equalizer is implemented as a discrete-time FIR filter applied to the sampled signal $s(t)$. Let $h_n, n = 1, ..., N$ denote the unknown impulse response coefficients of the channel equalizer, and $s_n, n = 1, 2, ..., M$ denote the corrupted drive signal, where M is, in general, much greater than N. The compensated drive signal $\hat{x}_n, n = N, ..., M$ is the result of convolving s_n with h_n. Algebraically, this operation can be conveniently written as

$$\begin{bmatrix} \hat{x}_N \\ \hat{x}_{N+1} \\ \vdots \\ \hat{x}_M \end{bmatrix} = \begin{bmatrix} s_1 & \cdots & s_N \\ s_2 & \cdots & s_{N+1} \\ \vdots & \cdots & \vdots \\ s_{M-N+1} & \cdots & s_M \end{bmatrix} \begin{bmatrix} h_N \\ \vdots \\ h_2 \\ h_1 \end{bmatrix}$$

or, using vector notation, more compactly as

$$\hat{\mathbf{x}} = S\mathbf{h} \ . \qquad (4)$$

If the receiver has synchronized to the transmitter with $\hat{\mathbf{x}}$ as it's input, then the receiver's output should be approximately equal to $\hat{\mathbf{x}}$. Specifically, synchronization requires that $\hat{\mathbf{x}} \approx \mathbf{r}$, where the vector $\mathbf{r}$ is used to denote the receiver output, *i.e.* $\mathbf{r} = (r_N, ..., r_M)^T$. Because the receiver output, $\mathbf{r}$, is a function of the channel equalizer impulse response, $\mathbf{h}$, we will use the notation $\mathbf{r}(\mathbf{h})$ to clearly indicate this dependency.

To design the equalizer to minimize the mean-square synchronization error, we determine $\mathbf{h}$ to minimize the quadratic cost function

$$J = (S\mathbf{h} - \mathbf{r}(\mathbf{h}))^T(S\mathbf{h} - \mathbf{r}(\mathbf{h})). \qquad (5)$$

A straightforward approach is to apply the gradient descent iteration

$$\mathbf{h}_{i+1} = \mathbf{h}_i - \gamma_i \frac{\nabla J}{||\nabla J||} \ . \qquad (6)$$

This approach involves computing the gradient of J at the i^{th} iteration of $\mathbf{h}$, say $\mathbf{h}_i$, and then updating the estimate of $\mathbf{h}$ by moving it in the direction of steepest descent of J.

The gradient of J is straightforward to calculate and is given by

$$\nabla J = 2(S^T - \nabla\mathbf{r}(\mathbf{h}_i))(S\mathbf{h}_i - \mathbf{r}(\mathbf{h}_i)) \ , \qquad (7)$$

where $\nabla\mathbf{r}(\mathbf{h}_i)$ denotes the Jacobian of $\mathbf{r}(\mathbf{h})$ evaluated at $\mathbf{h}_i$. This matrix is easily estimated numerically by perturbing the various components of $\mathbf{h}_i$ and measuring the resulting change in the receiver output.

There are two additional and important issues to consider when applying the proposed gradient descent algorithm. First, it is important to determine an appropriate step size, γ_i, at each iteration. We employed the "golden section search" algorithm to efficiently determine the step size which produces the largest possible change in J at each iteration. The second issue involves determining a good initial estimate of $\mathbf{h}$. One approach for obtaining a good initial estimate is to employ the coarse estimation procedure described in Section 2.1. Below, we illustrate this approach with a simple numerical experiment.

3. NUMERICAL EXPERIMENTS

For illustration we use for the chaotic system components in figure 1, the "scaled" Lorenz transmitter equations [2, 12]

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= rx - y - 20xz \\ \dot{z} &= 5xy - bz \ , \end{aligned}$$

and synchronizing receiver equations

$$\begin{aligned} \dot{x}_r &= \sigma(y_r - x_r) \\ \dot{y}_r &= rs(t) - y_r - 20s(t)z_r \\ \dot{z}_r &= 5s(t)y_r - bz_r \ . \end{aligned}$$

With $s(t) = x(t)$, the receiver will rapidly synchronize to the transmitter from any set of initial conditions.

3.1. Channel Gain Compensation

In this experiment, the Lorenz equations were numerically integrated using a fourth-order Runge-Kutta method with a fixed step size of .005. The corresponding sampling period on the received signal is $T = .005$.

We first assume that the channel imposes only a constant or time-varying gain on the transmitted drive signal $x(t)$, *i.e.* $C(s)$ in Fig. 1 is a gain G. If G differs from unity, then synchronization deteriorates, as illustrated Fig. 2. Specifically, Fig. 2 shows the synchronization error power P_e with no channel compensation, *i.e.* with $H(s) = 1$, and for constant channel gain or attenuation for a range of channel gain variation. As we expect, the synchronization error power is essentially zero for $G = 1$ and rapidly increases as the channel introduces non-unity gain or attenuation.

From Fig. 2 we see that the synchronization error power is unimodal for a range of gains around unity. As discussed in [11] a compensating gain can be obtained by minimizing P_e. This assumes that an initial estimate of the gain is within the range in Fig. 2 which is unimodal and includes $G = 1$ and for which the slope of the curve is sufficiently high to permit a reasonable gradient search for the gain.

A reasonable initial estimate of the gain can be obtained by utilizing the fact that P_x, the power in the chaotic drive signal at the input to the channel, can be expected to be independent of the specific sample path. Empirical measurement of P_x over a range of initial conditions in the Lorenz

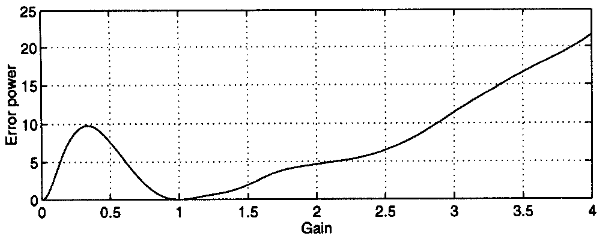

Figure 2. Synchronization error power vs. gain.

system with 500 independent trials and a time window of 800 sec resulted in an average value of 1.5978, a variance of 1.8×10^{-6}, a maximum of 1.6016 and a minimum of 1.5945. Based on this, a reasonable initial estimate of G is then taken as

$$G = \sqrt{\frac{P_{\hat{x}}}{P_x}} = \sqrt{\frac{P_{\hat{x}}}{1.5978}} . \tag{8}$$

A rectangular sliding window of duration Δ is used to estimate $P_{\hat{x}}$, *i.e.*

$$\hat{P}_{\hat{x}}(t) = \frac{1}{\Delta}\int_{t-\frac{\Delta}{2}}^{t+\frac{\Delta}{2}} \hat{x}^2(\tau)d\tau \tag{9}$$

In Fig. 3 we show a representative example of compensation for a constant gain $G = 3$. In this figure we show the synchronization error when the compensating gain is the reciprocal of the initial gain estimate in Eq. (8). We also show on the same graph the synchronization error when for each time window, a gradient search is applied to minimize P_e, starting with the initial estimate in Eq. (8). A 20-second sliding window was used to estimate $P_{\hat{x}}$. In Fig. 3(a) the initial error represents the error transient prior to synchronization. In Fig. 3(b) we show the error on an expanded scale for a 10-second interval after the initial transient. Clearly the gradient search following the initial gain estimate results in significant reduction in synchronization error.

3.2. Channel Frequency Response Compensation

In this experiment, the Lorenz equations were numerically integrated using a fourth-order Runge-Kutta method with a fixed step size of 0.01. The corresponding sampling period on the received signal is $T = 0.01$.

In figure 4, we show the averaged power spectrum of the transmitted chaotic signal samples, x_n. If the channel is nearly ideal, then $s_n \approx x_n$. In this case, s_n can be used to synchronize the receiver system. If the channel is far from ideal, however, the receiver will, most likely, not synchronize with s_n as it's input. To demonstrate this effect, we modeled the deterministic part of the channel by a low-order lowpass filter; additive white Gaussian channel noise, at a chaos-to-noise ratio level of 30 dB, was used to model the random channel components. The transfer function of the lowpass channel filter is given by

$$C(z) = \frac{-0.5 + z^{-1}}{1 - .8z^{-1}} .$$

Figure 5 shows the spectral magnitude and group delay of $C(z)$. From these plots, we see that $C(z)$ will alter the amplitude, bandwidth, and group delay of the chaotic drive signal. When the uncompensated received signal, s_n is used

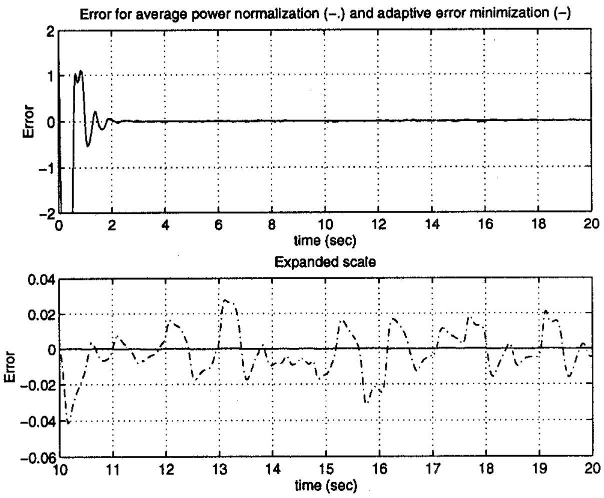

Figure 3. Receiver's synchronization error for average power normalization and adaptive error minimization.

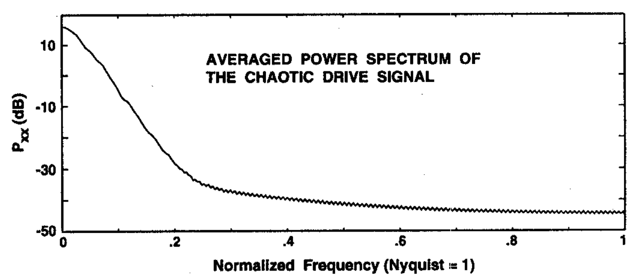

Figure 4. Power spectrum of the chaotic drive signal x_n.

as the input to the receiver, the chaos to error ratio is only 2 dB.

To apply the spectral ratio equalization approach, described in Section 2.1, we computed an averaged power spectrum of s_n using a 50-sec data window and then applied equation (3) to estimate the magnitude response of the spectral ratio channel equalizer. The filter transfer function or, equivalently, the impulse response was then estimated via spectral factorization methods. The resulting channel equalizer was implemented as an eight-point finite-impulse-response (FIR) filter; it's coefficients are approximately given by

$$\mathbf{h}_{sr} = [.19, .13, .09, .05, .01, -.01, -.02, -.02] .$$

We then determined the corresponding MSE equalizer coefficients. This was done by using $\mathbf{h}_{sr}$ as a starting point in the iterative optimization algorithm discussed in Section 2.2. After several iterations (15), the algorithm converged to

$$\mathbf{h}_{mse} = [.24, .17, .12, .07, .01, -.03, -.07, -.10] .$$

There are significant differences between the spectral ratio and MSE equalizers from the viewpoint of accurate channel equalization. To see this, consider figure 6 which shows the

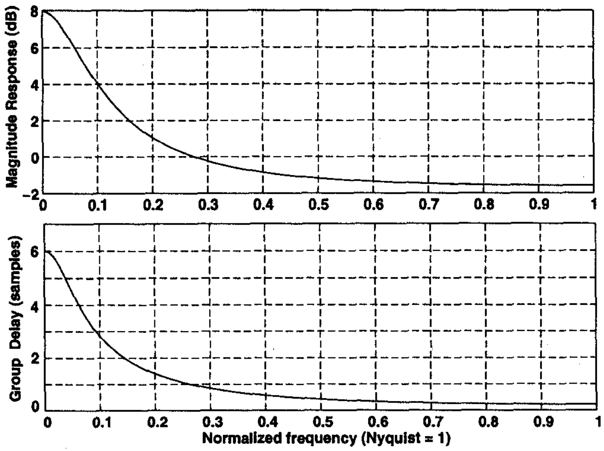

Figure 5. Spectral magnitude and group delay of a non-ideal channel.

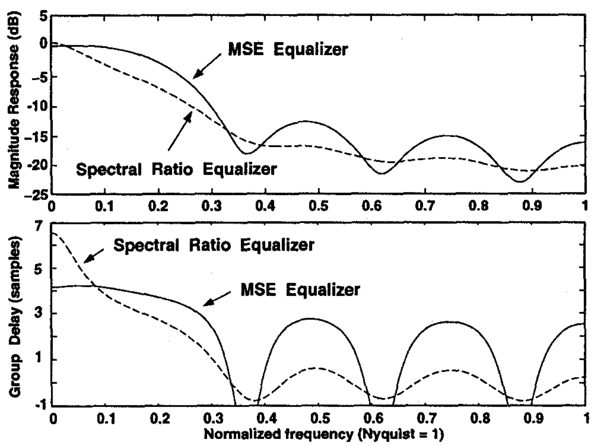

Figure 6. Compensated channel frequency response.

spectral magnitude and group delay of $C(z)H_{sr}(z)$ (dashed line) and $C(z)H_{mse}(z)$ (solid line). The MSE equalizer clearly does a much better job of equalizing the channel over the passband of the chaotic drive signal – the compensated channel has approximately unity gain and nearly constant group delay over the range of normalized frequencies from 0 to 0.2. Note also that both equalizers tend to attenuate high-frequency channel noise.

In figure 7, we compare the synchronization error resulting from application of the spectral ratio (dashed) and MSE (solid) equalizers. While the spectral ratio equalizer has improved the receiver's synchronization considerably, the chaos-to-error ratio is only 16 dB – not enough for some applications involving self-synchronizing chaotic systems. The performance of the MSE equalizer is significantly better; the resulting chaos-to-error ratio is about 33 dB. This performance can be improved upon further by using more coefficients in the equalizer, or by using a longer data window to estimate the coefficients.

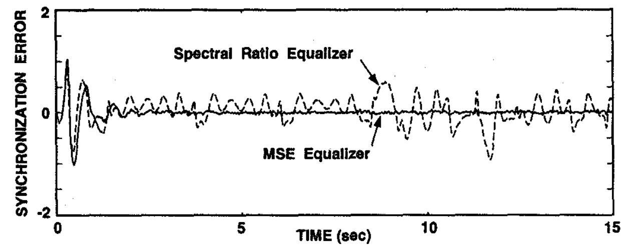

Figure 7. Synchronization error using: (a) spectral ratio equalizer; and (b) MSE equalizer.

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